CURRENT LIQUID CRYSTAL DISPLAY TECHNOLOGIES

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Chapter 1. Introduction

1.1 Displays:

Displays are the conduits to the information superhighway. Monitors, TV, sign boards, etc

Display requires knowledge in materials science, optical physics and electronics.

Progress in each area can affect the performance of the display. e.g. new LC materials, new polarizer, new electronic driving schemes, new optical modes for LCD.
### 1.2 Various kinds of displays

<table>
<thead>
<tr>
<th>Information displays:</th>
<th>Fixed displays</th>
<th>Electronic displays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Emissive</td>
<td>Non-emissive, light control</td>
</tr>
</tbody>
</table>

**Electronics displays:**
- Emissive
- Non-emissive, light control

**Emissive displays:**
- Neon light
- Cathode ray tube (CRT)
- Light emitting diode (LED)
- Electroluminescent display (ELD)
- Plasma display panels (PDP)
- Vacuum fluorescent display (VFD)
- Field emission display (FED)
- Flat CRT
- Organic ELD (OELD)
- Organic LED (OLED)

**Non-emissive display:**
- Mechanical displays
  - Liquid crystal display (LCD)
  - Digital mirror display (DMD)
  - Electrochromic display (ECD)
  - Electronic ink (e-ink)
  - Liquid crystal light valve (LCLV)
1.3 The display market:

Display classification:

Direct view: All of the above technologies

Projection:
Based on LCLV or CRT

Front projection
Rear projection
1.4 Cathode Ray Tube:

The deflection coils may be replaced by electrostatic fields.

The einzel lens is like optical lens for light.

Phorsphor efficiency: near 90%.
Chapter 2. Basic structure of a liquid crystal display:

Cell gap = 2 - 20 microns

Other materials used but not shown:

- Silver dots for connecting top ITO ground to bottom glass plate
- Epoxy perimeter seal
- Epoxy end seal
- Dopant for liquid crystal
- PI adhesion promoter
- Light diffuser/reflect
- Connectors to electronics
LCD operates by polarization manipulation of light using electrical pulses. Polarization change => brightness changes.

The effect of the applied voltage is first felt by the LC molecules, which rotate and align themselves in the electric field. This alignment change will then affect the optical properties and the polarization of the light passing through it.

Common model of a TN display:

![Diagram of TN display](image)

Figure 25. Principle of operation of a twisted nematic display in the *normally white* mode. The local optic axis undergoes a continuous 90° twist in the unactivated state (left), whereas it is predominantly parallel to the electric field in the activated state (right). (Figure adapted from reference 37)

We shall see that this picture is slightly inaccurate later!

Need to understand several things for the basic operation of LCD:

1. What is liquid crystal?  
   Phase transition
2. How can we align liquid crystals?
   Boundary conditions

3. How does an applied voltage affect the alignment?
   Elasticity, dielectric anisotropy

4. How does the alignment affect the optics?
   Optics of anisotropic and electro-active materials

For further understanding and design of LCD:

1. Flow dynamics of LC – response time

2. Optical modeling of LCD

3. Optical modeling with additional optical elements such as retardation films

**Different ways to view a LCD:**

1. **Transmittive:** The LCD is viewed in transmission. This is often the case for LCD with backlighting.
There is no reflector.

2. **Transflective**: This is the most popular. The rear reflector is often a diffusive type reflector to eliminate mirror effects.

![Diagram of Transflective LCD]

Ambient light or front lighting is used.

This transflective LCD is simply 2 transmittive LCD in tandem.

3. **Reflective**: In a truly reflective LCD, there is only one front polarizer. This is a new development with HKUST being one of the active participants.
The advantages are obvious. There is one less polarizer and also the reflector can be placed inside the LC cell.
3.1 Properties of Liquid Crystals:

A liquid that has partial crystalline structure.

Ordinary liquid: molecules are randomly oriented (amorphous)

Perfect crystal: all atoms are fixed at regular intervals. Only in solids (crystalline).
Usual behavior of liquid:

Liquid crystal material:

Melting temperature $T_m$

Clearing temperature $T_{cl}$

Liquid crystal is useful only between $T_m$ and $T_{cl}$. The typical useful range is between $-20^\circ\text{C}$ and $80^\circ\text{C}$. Can be as wide as $-30^\circ\text{C}$ to $120^\circ\text{C}$
3.2 Classification of liquid crystals:

Thermotropic: State of the liquid is determined by temperature. All liquid crystal displays are of this type.

Lyotropic: State of the liquid is determined by concentration. Useful in biological systems.

Amphitropic: Both temperature and concentration are important.

Shapes of the liquid crystal molecule:

Calamitic: Rod shape. All common LCDs are of this type.

Discotic: Disk like shape.

Phases of liquid crystals:

Nematic (N): All the LC molecules point in the same direction. All common LCD are of this type.

Smectic (Sm): The LC molecules point in the same direction and form layers.

Cholesteric (Ch): Twist angle between molecules. Same as chiral nematic.

Structures of the liquid crystal “crystal”: 

In a perfect crystal, all the molecules are fixed in space and orientation – no freedom
In a liquid, all the molecules are free to move and rotate – complete freedom

Nematic LC are fixed in orientation, free in translation

Smectic LC are fixed in orientation, fixed in z direction motion, and somewhat fixed in the x-y plane.

Fig. 1.1. Illustration of three thermotropic liquid crystals: nematic, smectic and cholesteric. (Redrawn from Lister and Birgeneau, 1982.)
Smectic B (SmB) forms a hexagonal structure in the x-y plane. (almost like a perfect crystal)

Chiral Smectic C (SmC*) is also called ferroelectric LC. More on that later.

Structurally, cholesteric = chiral nematic (N*) = twisted nematic

TN LCD = twisted nematic LCD
STN LCD = supertwisted nematic LCD
(All nematic.)

Chiral = twist. Can be induced by chiral dopants, or intrinsic in the LC molecule itself.

Whether a cholesteric is called a TN or STN or SSTN (Ch) depends on the natural pitch.

\[
\begin{array}{c}
\downarrow \\
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\uparrow \\
\downarrow \\
\uparrow \\
\end{array}
\]

TN pitch – 100 µm
STN pitch – 10 µm
Ch pitch – 1 µm
Common LCDs are made with nematic, thermotropic and calamitic (rod shape) liquid crystals.

Examples of common LC materials:

MBBA (p-methoxybenzylidene-p’-butylaniline)

\[ \text{CH}_3\text{O} - \text{C}=\text{N} - \text{C}_4\text{H}_9 \]

However, most nematic LCDs are doped to have a twist structure. TN, STN.

For any LC, we can describe the orientation of the molecules by the director \( \mathbf{n} \). For the simplest case

\[ \mathbf{n} = \mathbf{n}(z) \]

where \( z \) is distance perpendicular to the LC cell. The director \( \mathbf{n} \) is fully defined by the polar angle \( \phi \) and the azimuthal angle \( \theta \). For a LC cell, the tilt angle is defined as \( 90 - \theta \). It is the angle between the director and the x-y plane.

### 3.3 Order Parameter:

For nematic LC, we can define an order parameter \( S \) to describe the degree of ordering:

\[ S = \frac{1}{2} \langle 3 \cos^2 \theta - 1 \rangle \]
where we have defined the direction of the LC as the z-axis. Note that this is different from a LC cell. The averaging over $\phi$ is zero. $S$ is a measure of randomness of $n$. $n$ is distributed about the z-axis in a cone. Therefore

$$S = \frac{1}{2} \int_0^\pi (3 \cos^2 \theta - 1) f(\theta) \sin \theta d\theta / \int_0^\pi f(\theta) \sin \theta d\theta$$

where $f(\theta)$ is the distribution function of $\theta$.

Check: if $f=$constant, i.e. the director is randomly distributed,

$$S = 0$$

If $f(\theta) = \delta(0)$, i.e. the director is perfectly pointing in the same direction,

$$S = 1.$$

Maier-Saupe Theory shows that in general

$$S = \left(1 - \frac{yT}{T_{cl}}\right)^\beta$$
where \( y \) is of the order of 0.98 and \( \beta \) is in the range of 0.13-0.18. Hence the \( S \) for all LC are more or less the same in \( T \) dependence if plotted against normalized temperature \( T/T_{cl} \).

The following is the case for M15 and M21.

3.4 LC molecular structure

Most LC molecules are of the structure:

\[
Y - A - X - B - Z
\]
Where R, Z, X are radical groups, and A, B are aromatic rings such as benzene. Sometimes X may be missing. LC with different physical properties can be made by varying these groups.

\[
\begin{array}{ccc}
A, B & X & Y, Z \\
\text{benzene} & -\text{CH}=\text{N}- & -\text{R}(\text{alkyl}) \\
\text{cyclohexane} & -\text{N}=\text{N}- & -\text{RO} \\
\text{double benzene} & -\text{C})-\text{S}- & -\text{RCO} \\
& -(\text{CH}_2)_n- & -\text{COOH} \\
& -\text{CH}_2-\text{O}- & -\text{CN} \\
& & -\text{Cl} \\
& & -\text{NO}_2 \\
& & -\text{I} \\
& & -\text{F} \\
& & -\text{R}^* \\
\end{array}
\]

e.g. one of the early LC found is 5CB

\[
R_5 - \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} - \begin{array}{c}
\text{ } \\
\text{ } \\
\text{ } \\
\end{array} - \text{CN} \quad \text{(24 – 35.1}\degree\text{C})
\]

MBBA: p-methoxybenzylene-p’-butylaniline
EBBA: p-ethoxybenzylene-p’-butylaniline

X: -CH = N –
Y: CH\text{$_3$}O or C\text{$_2$}H\text{$_5$}O
Z: C\text{$_4$}H\text{$_9$}
All LC are transparent in the visible wavelengths.

Transmission \[ T = 10^{-OD} = e^{-\alpha d} \]

3.5 LC Mixtures and Phase Diagrams:

Usually, the liquid in a LCD is not a pure material. Several LCs are usually mixed together to get the right threshold voltage, clear point, birefringence, anisotropy etc.

Whenever 2 components are mixed together, there will be a phase diagram.
Phase diagram of a mixture of two nematogen (Blinov pg 20)

One can always mix different LCs to get a wider working temperature range if the two mixtures form a eutectic. Another example of a eutectic is lead-tin solder:

Melting temp of Pb and Sn are 327°C and 231°C respectively. But the melting temp of Pb-Sn solder is 215°C.

For LC, mixture is a must to extend the working temperature range.

Example:

Pure MBBA: LC from 22-47°C
Pure EBBA: LC from 37-80°C

60:40 MBBA:EBBA can be a LC from −10°C to 60°C!
In addition to wider temperature range, LCs are mixed to produce the desired threshold voltage and birefringence. For STN, a 4 bottle system is used to optimize the mixture. More on that later.

Other phase transitions: some compounds may have more than one LC phase. e.g.

\[ C_8H_{17}O - \text{NON} - OC_8H_{17} \]

Crystal ---> S\textsubscript{c} ----> N -----> I

79°  107°  125°
See Merck catalog to get an idea of the kinds of LC materials are available commercially.

An example of the data given in Merck catalog:

<table>
<thead>
<tr>
<th>Mixture:</th>
<th>MLC-6290-000</th>
<th>MLC-6290-100</th>
<th>MLC-6422</th>
<th>MLC-6438-000</th>
<th>MLC-6438-100</th>
<th>MLC-6457-000</th>
<th>MLC-6457-100</th>
</tr>
</thead>
<tbody>
<tr>
<td>S → N [°C]</td>
<td>&lt;40</td>
<td>&lt;40</td>
<td>&lt;40</td>
<td>&lt;40</td>
<td>&lt;40</td>
<td>&lt;40</td>
<td>&lt;40</td>
</tr>
<tr>
<td>Clearing point [°C]</td>
<td>+104</td>
<td>+104</td>
<td>+93</td>
<td>+89</td>
<td>+89</td>
<td>+90</td>
<td>+90</td>
</tr>
<tr>
<td>Viscosity [mm²s⁻¹]</td>
<td>20 °C</td>
<td>20</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td>17</td>
<td>22</td>
</tr>
<tr>
<td>Viscosity [mm²s⁻¹]</td>
<td>0 °C</td>
<td>63</td>
<td>53</td>
<td>68</td>
<td>70</td>
<td>52</td>
<td>72</td>
</tr>
<tr>
<td>Viscosity [mm²s⁻¹]</td>
<td>−20 °C</td>
<td>330</td>
<td>260</td>
<td>530</td>
<td>470</td>
<td>290</td>
<td>500</td>
</tr>
<tr>
<td>Viscosity [mm²s⁻¹]</td>
<td>−30 °C</td>
<td>1100</td>
<td>800</td>
<td>2600</td>
<td>1970</td>
<td>1090</td>
<td>2040</td>
</tr>
<tr>
<td>Viscosity [mm²s⁻¹]</td>
<td>−40 °C</td>
<td>5800</td>
<td>3700</td>
<td>22000</td>
<td>11660</td>
<td>5670</td>
<td>15000</td>
</tr>
<tr>
<td>n₀ (589nm, 20 °C)</td>
<td>1.6095</td>
<td>1.6124</td>
<td>1.7072</td>
<td>1.6545</td>
<td>1.6574</td>
<td>1.6666</td>
<td>1.6697</td>
</tr>
<tr>
<td>Δn (589nm, 20 °C)</td>
<td>+0.1200</td>
<td>+0.1198</td>
<td>+0.1978</td>
<td>+0.1503</td>
<td>+0.1502</td>
<td>+0.1600</td>
<td>+0.1602</td>
</tr>
<tr>
<td>Δε (1 kHz, 20 °C)</td>
<td>+8.8</td>
<td>+3.5</td>
<td>+9.9</td>
<td>+4.7</td>
<td>+10.2</td>
<td>+4.9</td>
<td></td>
</tr>
<tr>
<td>ε∥ (1 kHz, 20 °C)</td>
<td>12.8</td>
<td>6.9</td>
<td>-</td>
<td>14.3</td>
<td>8.3</td>
<td>14.6</td>
<td>8.6</td>
</tr>
<tr>
<td>Twist angle = 90°</td>
<td>1.00</td>
<td>1.00</td>
<td>.96</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>d·Δn [µm]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V₁₀[V]</td>
<td>1.90</td>
<td>2.98</td>
<td>2.55</td>
<td>1.77</td>
<td>2.50</td>
<td>1.83</td>
<td>2.57</td>
</tr>
<tr>
<td>V₅₀[V]</td>
<td>2.16</td>
<td>3.37</td>
<td>2.92</td>
<td>2.02</td>
<td>2.84</td>
<td>2.11</td>
<td>2.94</td>
</tr>
<tr>
<td>V₉₀[V]</td>
<td>2.60</td>
<td>4.03</td>
<td>3.60</td>
<td>2.45</td>
<td>3.42</td>
<td>2.59</td>
<td>3.60</td>
</tr>
<tr>
<td>(V₅₀/V₅₀ - 1)-100 [%]</td>
<td>13.68</td>
<td>13.09</td>
<td>14.29</td>
<td>14.12</td>
<td>13.60</td>
<td>15.03</td>
<td>14.62</td>
</tr>
<tr>
<td>(V₉₀/V₅₀ - 1)-100 [%]</td>
<td>36.84</td>
<td>35.23</td>
<td>40.90</td>
<td>38.14</td>
<td>36.80</td>
<td>41.26</td>
<td>40.35</td>
</tr>
<tr>
<td>Temp. dep. [m%°C](0 - 40 °C)</td>
<td>2.12</td>
<td>2.87</td>
<td>-</td>
<td>4.75</td>
<td>5.12</td>
<td>5.12</td>
<td>5.75</td>
</tr>
<tr>
<td>Temp. dep. [m%°C](0 - 40 °C)</td>
<td>.11</td>
<td>.10</td>
<td>.27</td>
<td>.21</td>
<td>.28</td>
<td>.22</td>
<td></td>
</tr>
<tr>
<td>M 20</td>
<td>1.85</td>
<td>1.74</td>
<td>1.88</td>
<td>1.78</td>
<td>1.92</td>
<td>1.83</td>
<td></td>
</tr>
<tr>
<td>M 0 - 40</td>
<td>1.93</td>
<td>1.81</td>
<td>2.09</td>
<td>1.93</td>
<td>2.15</td>
<td>1.99</td>
<td></td>
</tr>
<tr>
<td>M² 0 - 40</td>
<td>1.46</td>
<td>1.37</td>
<td>1.57</td>
<td>1.45</td>
<td>1.58</td>
<td>1.47</td>
<td></td>
</tr>
<tr>
<td>HTP (S-811) [1/µm]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>HTP (CB15) [1/µm]</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

format: L
Experimental observation of phase transition

Light scattering from the LC: in the nematic state, without boundaries, the LC forms domains from isotropic to LC state. It is because ordering is spontaneous and localized.

Small domains => scattering (hence milky appearance)

This is an actual experimental data. One can see the sharpness of the transition and the huge dynamic range.
Another kind of mixture is with chiral dopants. Commercial LCs are doped to give it a twist.

3.6 Twisted nematic and cholesteric LC:

For cholesterics,

\[ \mathbf{n} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta) \]

where \( \phi = qz \)

and \( \theta = \text{constant} \).
The tilt angle is constant => $\theta$ is constant.

The pitch is defined to be the distance when the twist is $2\pi$.

Therefore $2\pi = qp$

Or $q = \frac{2\pi}{p}$

For a cell with thickness $d$, the total twist angle within the cell, $\phi_o$, is therefore:

$$\phi_o = \frac{2\pi}{p}d$$

or

$$\frac{d}{p} = \frac{\phi_o}{2\pi}$$

The $d/p$ ratio of a LC cell is a very important parameter in determining its properties. Here $p$ is the natural pitch. In actual LC cells, the actual pitch and the natural pitch may not be the same. This leads to elastic stress which sometimes is good for the operation.

If there is no stress, the $d/p$ ratio of a $90^\circ$ twist cell is 0.25. The $d/p$ ratio of a $240^\circ$ twist cell is 0.67.

For a cholesteric cell, there can be several twists inside the cell. e.g. 10. Therefore $d/p = 10$.

If there is more than one twist inside the cell, there will be **Bragg** reflection. This is because the cell acts as N layers of
materials of different refractive index, similar to x-ray diffraction

For a certain resonant wavelength, there will be maximum reflection

\[ 2n \left( \frac{p}{2} \right) \sin \theta = \lambda \]

Hence, at near normal incidence, the resonant wavelength is given by

\[ \lambda = np \]

This is quite dramatic if \( \lambda \) is near the optical wavelength. The LC cell will appear to have bright color.
This can be the principle of a temperature sensor (liquid crystal thermometer). If the pitch is strongly dependent on temperature, the color of the LCD will change as a function of temperature. e.g. choleyl carbonate
Chapter 4. Basic Optics:

Light has 2 important properties that are relevant to vision: colour and brightness. They belong to the realm of photometry which will be discussed later. Here we review some optical properties of isotropic and anisotropic materials.

4.1 Colour:

Colour is characterized by wavelength. Symbol of wavelength is $\lambda$.

<table>
<thead>
<tr>
<th>Colour</th>
<th>wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>600-650 nm</td>
</tr>
<tr>
<td>Green</td>
<td>520-570 nm</td>
</tr>
<tr>
<td>Blue</td>
<td>450-490 nm</td>
</tr>
</tbody>
</table>

4.2 Origin of the refractive index – isotropic materials

Dielectric response: $E \rightarrow P \rightarrow D$

Lorentz model:

$$m \frac{d^2 x}{dt^2} + m\Gamma \frac{dx}{dt} + m\omega_o^2 x = -qE e^{j\omega t}$$

where $\omega_o$ is some resonance frequency and $\Gamma$ is the damping coefficient of that resonance.
This leads to the solution of $x$ and the polarization $p=-qx$.

\[ p = \alpha E \]  
where $\alpha$ is the polarizability

Then \[ P = Np \]

\[ D = \varepsilon_o E + P = \varepsilon E \]

gives

\[ \varepsilon = \varepsilon_o + \frac{Nq^2 / m}{\omega_o^2 - \omega^2 + j\omega\Gamma} \]

Each resonance will give a term like this. Therefore

\[ \varepsilon = \varepsilon_o + \sum_i \frac{N_i q_i^2 / m_i}{\omega_{oi}^2 - \omega^2 + j\omega\Gamma_i} = \varepsilon' - j\varepsilon'' \]

Relative permittivity:  \[ \varepsilon_r = \varepsilon / \varepsilon_o \]

Index of refraction $n$:

\[ n = \sqrt{\varepsilon_r} = n' - jn'' \]
The real part of $n$ leads to phase velocity and phase retardation. The imaginary part leads to absorption.

\[ \lambda = \frac{c}{nV} \]

Also note that $\varepsilon$ at low frequency (DC-RF) is very different from $\varepsilon$ at optical frequencies. e.g. water has $\varepsilon = 81$ at DC and 2 at optical wavelengths.

Refractive index is usually defined for the optical wavelengths.
Every medium is characterized by a **refractive index** \( n \). For homogeneous medium, \( n \) is a constant. For example, glass has \( n=1.5 \). Many materials, such as LC and polarizers, are inhomogeneous, or anisotropic.

For light going from a material with refractive index \( n_1 \) to a medium with refractive index \( n_2 \), the refraction angle is given by **Snell’s law**:

\[
n_1 \sin \theta = n_2 \sin \phi
\]

For the case of \( n_1 > n_2 \), \( \phi = 90^\circ \) occurs at \( \theta = \theta_c \). This is called the critical angle for total internal reflection (TIR).

\[
\sin \theta_c = \frac{n_2}{n_1}
\]
e.g. for glass \((n = 1.5)\) to air, \(\theta_c = 41.8^\circ\). Therefore a right angle prism can be used as a high efficiency reflector.

There is a phase shift associated with TIR. For Fresnel reflection (medium 1 to medium 1), the phase shift is \(-180^\circ\). For TIR, the phase shift depends on the polarization:

\[
\tan \frac{\delta_s}{2} = \frac{\sqrt{n^2 \sin^2 \theta - 1}}{n \cos \theta}
\]

and \(\tan \frac{\delta_p}{2} = \frac{n \sqrt{n^2 \sin^2 \theta - 1}}{\cos \theta}\) for \(n_2 = 1\)

This phase shift is important in designing optical coatings.

The reflection and transmission coefficients are the ratio of the reflected and transmitted light. They depend on the polarization of the light.

**4.4 Polarization**
Light can be polarized or unpolarized. e.g. light bulb is unpolarized. Sun light is partially polarized. Laser beams can be totally polarized.

Unpolarized light can be made polarized by passing it through a polarizer.

Common forms of polarization: linear polarization and circular polarization.

**Linearly polarized light:**

\[ \mathbf{E} = xE_o \cos (\omega t - kz) \]

where \( k = \frac{2\pi n}{\lambda} \)

\( E_o = \) wave amplitude

\( \lambda = \) wavelength

\( \nu = \) frequency

In phasor notation

\[ E = xE_o e^{j\omega t - jkz} \]

This wave propagates in the z-direction and is polarized in the x-direction.
In general, the wavevector $\mathbf{k}$ determines the direction of propagation. The direction of $\mathbf{E}$ determines the polarization. Superposition of waves is also a wave.

Phase velocity of the wave

$$c = \frac{\omega}{k}$$

$$k = n k_0$$

How does one get polarized light? – Use polarizers
Polarizers:

(1) Absorptive polarizers: e.g. polaroid. Preferential absorption of light in one direction and transmission in the perpendicular direction. This is the most popular polarizer used in LCD. There will be more discussions about the polarizer material in a later section.

Randomly polarized light going through the polarizer becomes polarized in the direction of the polarizer.

![Diagram of polarizer with input light polarization and polarizer axis]

Linearly polarized light passing through the polarizer will be reduced in amplitude and will change direction of polarization:

![Diagram showing input light polarization and polarizer axis]
We should break the input light polarization into a component parallel and a component perpendicular to the polarizer axis. Only the component that is parallel to the polarizer axis will be passed through.

\[ T = \cos \theta \]

(2) Reflective polarizer: e.g. polarizing beam splitters (PBS)

PBS is used in LCD projection systems.

Circularly and elliptically polarized light:

The most general elliptically polarized light is formed by adding two coherent perpendicularly linearly polarized light with an arbitrary phase shift of \( \delta \)

\[ \mathbf{E} = x \mathbf{E}_a \cos (\omega t - kz) - y \mathbf{E}_b \cos (\omega t - kz + \delta) \]
This equation describes a right elliptically polarized wave. The tip of the E vector describes an ellipse if viewed from the front.

If $E_a = E_b$, and $\delta = 90^\circ$, then this equation describes a right circular polarized wave. It describes a right-handed corkscrew as the wave propagates. At a fixed $z$, the polarization (E vector) rotates clockwise.

4.5 Reflection and refraction of polarized light (Fresnel formulas):
Plane of incidence = plane containing the incident and reflected light beams = plane of the drawing

p-polarized light = linearly polarized light with electric field oscillating parallel to the plane of incidence

s-polarized light = linearly polarized light with electric field oscillating perpendicular to the plane of incidence

There are well-known formulas to calculate the reflectance of s- and p-polarized light. (**Fresnel formulas**).

Fresnel formulas:

\[
    r_s = \frac{-\sin(\theta - \phi)}{\sin(\theta + \phi)}
\]
\[ r_p = \frac{-\tan \left( \theta - \phi \right)}{\tan \left( \theta + \phi \right)} \]

\[ t_s = \frac{2 \sin \phi \cos \theta}{\sin(\theta + \phi)} \]

\[ t_p = \frac{2 \sin \phi \cos \theta}{\sin(\theta + \phi) \cos(\theta - \phi)} \]

These are ratios of electric fields. The reflection and transmission coefficients are defined by the ratio of the Poynting vectors. For reflection,

\[ R_s = \frac{\sin^2(\theta - \phi)}{\sin^2(\theta + \phi)} \]

and

\[ R_p = \frac{\tan^2(\theta - \phi)}{\tan^2(\theta + \phi)} \]

Graphs showing the reflection coefficient of s- (upper) and p-polarized (lower) light off glass (n=1.5):
Observations:

1. R=4% both s-pol and p-pol cases at normal incidence.

2. The s-pol case (bottom curve) goes to zero at 56°. This is called the polarizing angle, or the Brewster angle. At this angle, all the reflection is totally polarized, even if the incident light is unpolarized.
\[
\tan \theta_B = \frac{n_2}{n_1}
\]

e.g. for glass to air, \( \theta_B = 56.3^\circ \).

3. We can make use of this Brewster angle to make a polarizer:

3. This is also the principle of polaroid sunglasses. The glare reflected off the pool or the front of the car is horizontally polarized. So if the sunglass is vertically polarized, the glare will be absorbed.

4. One can play a lot of other tricks with light polarization.

4.6 **Birefringence (optical anisotropy):**

A material that has 2 refractive index is called birefringent. A retardation plate or a stretched piece of plastic is birefringent. A liquid crystal cell is birefringent, plus also having optical activity. Many crystals are birefringent.

A birefringent material has 2 refractive indices

\[ n_0 = \text{ordinary index} \]
\[ n_e = \text{extraordinary index} \]
\[ \Delta n = n_e - n_o = \text{birefringence} \]

Basically, light polarized in one direction will see a refractive index of \( n_o \). If it is polarized in the other direction, it will see a refractive index of \( n_e \). The fast axis sees the extraordinary index.

Note: LC has very large birefringence! \( \Delta n \) ranges from 0.05 to 0.22.

Refractive index of LC is wavelength dependent and temperature dependent:

Typical wavelength dependence:
Typical temperature dependence:
4.7 Optics of anisotropic materials:

(Reference: Saleh and Teich, Photonics)

$\varepsilon$ is a 3x3 tensor in general. All elements can be non-zero.

For isotropic materials, $\varepsilon$ is given by

$$
\varepsilon = \begin{pmatrix}
\varepsilon & 0 & 0 \\
0 & \varepsilon & 0 \\
0 & 0 & \varepsilon
\end{pmatrix} = \varepsilon_{I}
$$

Maxwell’s equations:

$$
\nabla \times H = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t}
$$

$$
\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t}
$$

give the wave equation:

$$
\nabla^2 E = \mu \varepsilon \frac{\partial^2 E}{\partial t^2}
$$

and the wave solutions:
\[ E = E_0 e^{j\omega t - jkz} \]

with phase velocity

\[
c = \frac{\omega}{k} = \frac{\lambda v}{n}
\]

For an anisotropic medium, there are three wave equations.

The dielectric constant in the principal axes is diagonal:

\[
\varepsilon = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix}
\]

The \( (x, y, z) \) axis corresponds to some symmetry axis in the medium. For LC, obviously, \( z \) will be along the LC director. Note that this \( z \)-axis is in the molecular frame. Need to transform to the lab frame later. The symmetry axis of the crystal is often called the c-axis, e.g. the director of LC. Now

\[
\begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix} = \begin{pmatrix}
\varepsilon_x & 0 & 0 \\
0 & \varepsilon_y & 0 \\
0 & 0 & \varepsilon_z
\end{pmatrix} \begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\]
We can substitute this into the Maxwell’s equations to obtain 3 wave equations:

\[ \nabla^2 E_x = \mu \varepsilon_x \frac{\partial^2 E_x}{\partial t^2} \]

\[ \nabla^2 E_y = \mu \varepsilon_y \frac{\partial^2 E_y}{\partial t^2} \]

\[ \nabla^2 E_z = \mu \varepsilon_z \frac{\partial^2 E_z}{\partial t^2} \]

So a wave propagating in the z-direction, polarized in the y-direction will have a k-vector

\[ k_y = \sqrt{\frac{\varepsilon_y}{\varepsilon_o}} k_o = n_y k_o \]

But in general, the wave may propagate not along the principal axes, and the polarization may not be along the principal axes. This problem can be solved quite easily for the case of a uniaxial material. LC is a uniaxial material. There is symmetry in the x-y axes (perpendicular to the LC molecule). If there is no such symmetry, we have the case of a biaxial material which can be messy to solve.

For a uniaxial material, let \( \varepsilon_x = \varepsilon_y \). Then we can use the index ellipsoid to solve this problem.
The index ellipsoid is given by

\[
\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1
\]

For biaxial material, all \( n \)'s are different. For uniaxial material

\[
n_x = n_y = n_o
\]

\[
n_z = n_e
\]

\( n_o \) is called the \textit{ordinary} index and \( n_e \) is called the \textit{extraordinary} index.
The general procedure for obtaining the wave propagating modes for an uniaxial material is as follows. Let the wave vector make an angle \( \theta \) to the z-axis. The plane perpendicular to \( \mathbf{k} \) will intercept the index ellipsoid. The interception itself is an ellipse, with a major and a minor axis. The minor axis is always along \( \mathbf{x} \). The major axis depends on \( \theta \).

Then the eigen-wave propagating in this medium will have \( \mathbf{D} \) (hence polarization) either parallel to the major or the minor axis of this ellipse. The wave with \( \mathbf{E} \) along the major axis is called the **extraordinary** wave (e-wave). The wave with \( \mathbf{E} \) parallel to the minor axis is called the **ordinary** wave (o-wave). Note that the ordinary wave does not depend on \( \theta \).
The refractive index seen by the ordinary wave is the length of the minor axis, while the refractive index seen by the extraordinary wave is \( n_e(\theta) \).

\[
\frac{1}{n_e^2(\theta)} = \frac{\cos^2 \theta}{n_o^2} + \frac{\sin^2 \theta}{n_e^2}
\]

This is also called double refraction. Note that only e-waves and o-waves can propagate. NO other polarizations are allowed. For a general polarization wave coming into an uniaxial medium, one has to decompose it into a o-wave and an e-wave.

Note also that \( \mathbf{D} \) and \( \mathbf{E} \) are slightly different in direction for the e-wave. This leads to wave separation for a thick crystal. But for LCD, we can ignore it.

For a retardation film, the extraordinary axis (z-axis) is also called the fast axis or the c-axis.

**Birefringence**

\[
\Delta n = n_e - n_o
\]

Positive birefringent: \( \Delta n > 0 \).
Negative birefringent: \( \Delta n < 0 \).
4.8 Retardation film:

When light passes through a birefringent material, the e-wave and the o-wave accumulate different phase delay. This is called phase retardation. The retardation value of a retarder is defined as

\[ \delta = \frac{\pi d \Delta n}{\lambda} \]

where \( d = \) film or cell thickness

Retardation films are used in LCD for viewing angle enhancement and color compensation (dispersion compensation). It is also used in optics for polarization manipulation. A retardation plate can turn linear polarized light to circular polarized light and vice versa. It can also rotate the polarization of linearly polarized light.

The most common arrangement for the retardation film or plate is to have the fast axis on the plane of the sheet, and the input light at 45° to the fast axis.

Using this geometry, the input light has to be decomposed into e-wave and o-wave:

\[ E = x \frac{E_o}{\sqrt{2}} e^{-jkx} + y \frac{E_o}{\sqrt{2}} e^{-jky} \]

where we have taken the y-axis as the fast axis. At the output, the wave becomes
\[ E = \left( x \frac{E_o}{\sqrt{2}} e^{-jk_x d} + y \frac{E_o}{\sqrt{2}} e^{-jk_y d} \right) e^{-jkz + j\omega t} \]

We can take out the common phase and write this as

\[ E = \left( x \frac{E_o}{\sqrt{2}} e^{j\delta} + y \frac{E_o}{\sqrt{2}} e^{-j\delta} \right) e^{-jkz + j\omega t - j\frac{\pi(n_o+n_e)d}{\lambda}} \]

There are many applications of this formula:

1. Quarter-wave plate:

This is the case for \( \delta = \pi/4 \)

or \( d\Delta n = \lambda/4 \)

The output wave is given by

\[ E = xE_o \cos (\omega t - kz) + yE_o \sin (\omega t - kz) \]

which is a right-circularly polarized light. Therefore a \( \lambda/4 \) waveplate can turn linear polarization to circular polarization and vice versa.

Geometry: fast axis at 45° to the input polarization direction.
In general, if the polarizer is not at 45° to the fast axis, we get an elliptically polarized light output.

2. Half-wave plate:

This is the case for \( \delta = \pi/2 \)

or \( d\Delta n = \lambda/2 \)

Geometry: Input linear polarization direction is at angle \( \theta \) to the fast axis.
Input wave is decomposed into e-wave and o-wave:

\[
E = x E_O \cos \theta e^{-jkx\hat{z}} + y E_o \sin \theta e^{-jky\hat{z}}
\]

Output wave is given by

\[
E = x E_O \cos \theta e^{-jkx\hat{z}} - y E_o \sin \theta e^{-jky\hat{z}}
\]

Therefore, the output is rotate by 2\(\theta\), or reflected about the fast axis. As the half wave plate is rotated, the output polarization also rotates (at twice the angle).

3. Variable attenuator:

Standard geometry:
- cross polarizer/analyzer (or cross polarizers)
- fast axis at 45°
Transmittance through the retardation film in the standard geometry:

\[ T = \sin^2 \frac{\pi d \Delta n}{\lambda} = \sin^2 \delta \]

For a LCD, \( \Delta n \) can be controlled. This arrangement is the basis of the electrically controlled birefringence (ECB) display. More on that later.

4. Oblique angle of incidence:

Similar formulas can be obtained for the case that the fast axis is not on the plane of the waveplate. e.g. LC cell with a tilt angle, or wave incident at oblique angles. There will be Poynting vector walkoff and double refraction angles.

Double refraction:
2 cases: c-axis in the plane of incidence or perpendicular to plane of incidence. Let’s look at first case.

From Snell’s law,

\[ n_o \sin \theta_o = n_g \sin \theta_t \]

\[ n_e (90^\circ - \theta_e + \alpha) \sin \theta_e = n_g \sin \theta_t \]

where \( \alpha \) = pretilt angle. Simple case: let the pretilt angle = 0. i.e. the crystal axis is parallel to the x-axis on the glass surface.

Hence

\[ \frac{1}{n_e^2 (90^\circ - \theta_e)} = \frac{\sin^2 \theta_e}{n_o^2} + \frac{\cos^2 \theta_e}{n_e^2} \]
We can then solve for $\theta_e$ and get

$$\cot^2 \theta_e = \frac{n_e^2}{n_o^2} \left( \frac{n_o^2}{n_g^2 \sin^2 \theta_i} - 1 \right)$$

For example, if light is incident from air at $60^\circ$. Let $n_o = n_g = 1.5$ and $n_e = 1.7$. Then $\theta_i = \theta_o = 35.3^\circ$, and $\theta_e = 32^\circ$.

**Poynting vector walkoff:**

Since $\mathbf{D} = \varepsilon \mathbf{E}$

$\mathbf{D}$ and $\mathbf{E}$ will not be parallel if $\varepsilon$ is a tensor.

Also, since $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$ \hspace{1cm} $\mathbf{D} \perp \mathbf{H}$ and $\mathbf{D} \perp \mathbf{k}$

And $\nabla \times \mathbf{E} = \frac{\partial \mathbf{H}}{\partial t}$ \hspace{1cm} $\mathbf{H} \perp \mathbf{k}$ and $\mathbf{H} \perp \mathbf{E}$
Also \[ S = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H} \quad \mathbf{s} \perp \mathbf{H} \text{ and } \mathbf{s} \perp \mathbf{E} \]

Now from the \( \varepsilon \) tensor,

\[
\begin{pmatrix}
D_x \\
D_y \\
D_z
\end{pmatrix} =
\begin{pmatrix}
\frac{n_o^2}{2} & 0 & 0 \\
0 & \frac{n_o^2}{2} & 0 \\
0 & 0 & \frac{n_e^2}{2}
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
\]

Therefore, equating the \( x \) and \( z \) components give (there is no \( y \) component):

\[
D_e \cos \theta = n_o^2 E_e \cos (\theta - \alpha)
\]
\[
D_e \sin \theta = n_e^2 E_e \sin (\theta - \alpha)
\]

We can then solve for \( \alpha \):
\[ \tan \alpha = \frac{(n_e^2 - n_o^2) \tan \theta}{n_e^2 + n_o^2 \tan^2 \theta} \]

For \( \theta = 0^\circ \) and \( 90^\circ \), there is no beam walkoff. For \( \theta = 45^\circ \), there is max walkoff.

For example, \( n_o = 1.5 \), \( n_e = 1.7 \), \( \theta = 45^\circ \), can calculate \( \alpha = 7.1^\circ \). This is quite large. For LCD, this is not a big problem because the cell thickness is small.