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TOPICAL REVIEW

Controlling the speed of light pulses

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Abstract
Recent experiments have demonstrated that light pulses can be made to propagate 'superluminally' or, at the opposite extreme, to come to a complete stop. Some of the basic physics underlying these studies and their antecedents are reviewed.

1. Introduction

The most primitive notions of causality, together with the special theory of relativity, demand that information cannot be propagated with a velocity exceeding c, the speed of light in vacuum. But various velocities, such as the phase velocity of light, can exceed c without violating special relativity because they do not in general represent velocities of *signals*, or information. A persistent misconception is that the *group* velocity v_g of a light pulse cannot exceed c, or that the very concept of group velocity breaks down if $v_g > c$. Experiments, however, have demonstrated that the group velocity can exceed c, and can even be infinite, while remaining perfectly meaningful as the velocity with which a waveform propagates. At the other extreme, v_g can be much less than c or even zero. The purpose of this paper is to review some of the basic physics of 'superluminal' and highly subluminal (or zero) group velocities.

In the following section we briefly review some basic ideas about causality, reminding ourselves why superluminal communication is incompatible with the special theory of relativity and quantum theory. We also discuss briefly the Fermi model of photon propagation in vacuum. In section 3 we review some important aspects of the classic work of Sommerfeld, Brillouin, and others on causality in the classical theory of electromagnetic wave propagation. Sections 4–6 are devoted to the 'group velocity approximation' and related considerations in this theory, and the fact that the group velocity of a pulse can exceed c while remaining meaningful as a velocity of pulse propagation. Experimental observations of superluminal group velocities by Chiao *et al*, Wang *et al* and others are described, as well as earlier work along these lines. We attempt to clarify some of the misconceptions and controversies surrounding the concepts of group velocity, signals and energy propagation. In section 7 we discuss the limitations on the observability of superluminal group velocities due to quantum noise in amplifiers. In section 8 we turn our attention to the opposite extreme of highly *sub*luminal group velocities, and provide some of the background required for an understanding of recent experiments on ultraslow light as well as the complete halting and 'storage' of light. Because the 'superluminal' propagation of light pulses seems to attract more conceptual difficulties and controversies than the case of 'ultraslow' light, it was decided to devote more attention to it here.

2. Causality

The requirement that no signal can be transmitted with a velocity exceeding c is often referred to as 'Einstein causality', and can be inferred as follows.

Causality in its most basic form demands that 'the effect should not precede the cause'. Suppose an event at (x, t) were to cause an event at $(x + \Delta x, t + \Delta t)$ via some signal with velocity u. In some other reference frame with relative velocity v ($<c$) the time interval between the events is

$$\Delta t' = \frac{\Delta t - (v/c^2)\Delta x}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t(1 - uv/c^2)}{\sqrt{1 - v^2/c^2}}. \tag{1}$$

Then a superluminal signal velocity ($u > c$) implies there are v for which Δt and $\Delta t'$ have opposite signs: the temporal order of 'cause' and 'effect' would be different for different observers. Special relativity and causality thus forbid superluminal *signal* velocities. But it is instructive to consider examples where this fact is not immediately obvious, and we now consider a few such examples.

2.1. Forbidden filters

Consider a 'black box' that produces an output $F_{out}(t)$ that (a) depends linearly on the input $F_{in}(t)$ and (b) is time independent in the sense that a shift in time of the input produces the same shift in time of the output. Thus we write

$$F_{out}(t) = \int_{-\infty}^{\infty} \mathrm{d}t'\, G(t - t')F_{in}(t'), \tag{2}$$

or, in terms of the corresponding Fourier transforms, $f_{out}(\omega) = g(\omega) f_{in}(\omega)$.

Suppose further that there is no input until the time $t = 0$, so that for $t < 0$ there is complete destructive interference of the Fourier components of F_{in}. Causality then requires that there be no output before $t = 0$. Now suppose that our black box does nothing but absorb the frequency component ω. Then its output would be $F_{out} = F_{in} - F_{in,\omega}$, which does not vanish for $t < 0$—there would be an output before there were any input. *It must therefore be impossible to have a perfect filter, one that absorbs one frequency without affecting any other frequency*. Any realizable filter must also produce phase shifts in all the other Fourier components, in such a way that they interfere destructively for all $t < 0$, so that indeed there is no output before any input [1].

The mathematical expression of this requirement, of course, is a (Kramers–Kronig) dispersion relation between the absorptive and dispersive parts of a response function. No output before any input means that $G(t - t') = 0$ for $t < t'$, so that

$$g(\omega) = \int_{-\infty}^{\infty} \mathrm{d}\tau\, G(\tau)\mathrm{e}^{\mathrm{i}\omega\tau} = \int_{0}^{\infty} \mathrm{d}\tau\, G(\tau)\mathrm{e}^{\mathrm{i}\omega\tau}. \tag{3}$$

The simple but crucial point is that causality requires that the integral extends over only half the τ axis. Then, for $\tau > 0$, $g(\omega)$ is analytic in the upper half of the complex ω plane, and it is this analyticity that leads to the Hilbert-transform relations (i.e. dispersion relations) between the real and imaginary parts of $g(\omega)$ [2].

2.2. Causality and quantum theory

A quantum field[1] $\hat{\phi}(\boldsymbol{x}, t)$ can create or annihilate particles when it acts on a state $|\psi\rangle$. If $c^2(t-t')^2 - (\boldsymbol{x}-\boldsymbol{x}')^2 < 0$ the creation and annihilation events should not affect one another. Thus $\hat{\phi}(\boldsymbol{x}, t)\hat{\phi}(\boldsymbol{x}', t')|\psi\rangle = \hat{\phi}(\boldsymbol{x}', t')\hat{\phi}(\boldsymbol{x}, t)|\psi\rangle$; i.e., the commutator $[\hat{\phi}(\boldsymbol{x}, t), \hat{\phi}(\boldsymbol{x}', t')] = 0$ for spacelike separations. Any physical process that would violate this condition would be inconsistent with causality and must therefore be forbidden. Obviously this is a rather formal condition and, as in the classical case, it is instructive to consider explicit examples where the requirement of causality puts constraints on what physical processes are possible.

2.2.1. No superluminal signalling via EPR correlations. Part of the mystique surrounding Einstein–Podolsky–Rosen (EPR) correlations stems from the 'spooky action at a distance' they seem to imply. Consider the example of two photons in the entangled polarization state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H_A\rangle|V_B\rangle - |V_A\rangle|H_B\rangle), \tag{4}$$

where H and V denote 'horizontal' and 'vertical' polarization. Alice and Bob each observe one member of the photon pair: if Alice measures H (V) polarization, then the state of Bob's photon is immediately reduced to V (H). This sort of 'action at a distance' cannot be used for instantaneous *communication of information*, or signalling, because Alice cannot *choose* whether to 'send' a V or an H to Bob; she has a 50/50 chance of getting an H or a V herself.

She does, of course, have a choice as to polarization basis. She can, for example, use the circular polarization states $|R\rangle = (1/\sqrt{2})(|H\rangle - \mathrm{i}|V\rangle)$ and $|L\rangle = -(1/\sqrt{2})(|H\rangle + \mathrm{i}|V\rangle)$ instead of $|H\rangle$ and $|V\rangle$. In this basis we can write the state (4) equivalently as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|R_A\rangle|R_B\rangle - |L_A\rangle|L_B\rangle). \tag{5}$$

Thus, if Alice chooses to work in the H, V basis, her measurement reduces Bob's photon state to V or H, whereas if she chooses to work in the R, L basis, her measurement reduces Bob's photon state to R or L. However, her choice of basis cannot serve to send information to Bob because, given a single photon, Bob cannot distinguish between linear or circular polarization. That is, there is no device that can measure the polarization parameters of a single photon [3]. If such a device were possible, EPR correlations could be used for superluminal (instantaneous) communication.

Consider alternatively the density matrix ρ_B describing Bob's photon. Tracing over Alice's states, we obtain $\rho_B = \mathrm{Tr}_A[|\Psi\rangle\langle\Psi|] = (1/\sqrt{2}(|H_B\rangle\langle H_B| + |V_B\rangle\langle V_B|)$ in the H, V basis and $\rho_B = (1/\sqrt{2})(|R_B\rangle\langle R_B| + |L_B\rangle\langle L_B|)$ in the R, L basis. These are just different ways of writing the same density matrix, and so Alice's choice of whether to measure linear or circular polarization cannot affect Bob's measurements and therefore cannot be used to transmit information.

2.2.2. No cloning. But if Bob has *many* particles in the same state he can perform measurements to determine that state. For example, he can let N ($\gg$1) photons pass through polarization-dependent beam splitters and thereby determine with a high degree of accuracy whether a measurement of linear or circular polarization was made on Alice's EPR-correlated photon. In other words, if the single photon at Bob's end could be sent through an amplifier to produce a large number of photons in the same (arbitrary) state, it would be possible for Alice to communicate superluminally to Bob whether she measured linear or circular polarization [4].

[1] A caret ($\wedge$) is used to denote a quantum mechanical operator.

This superluminal communication scheme fails because photons cannot be perfectly copied: the amplifier will produce photons with polarization different from the incident photon by spontaneous emission—we cannot have stimulated emission without spontaneous emission, and therefore we cannot amplify an arbitrary polarization [5]. Much more generally, quantum theory does not permit the cloning of a single quantum [6, 7].

It is worth noting that this conclusion does not require that an optical amplifier be polarization dependent. It is possible for an amplifier to produce a final state that is independent of the polarization of the incident photon, but such an amplifier does not violate the no-cloning theorem: 'the essential element that prevents cloning is here seen to be spontaneous emission, rather than any dependence of amplifier gain on polarization' [8].

Glauber [9] has discussed in more detail the impossibility of superluminal communication using laser amplifiers, and has shown that amplification does not allow Bob to ascertain whether Alice measures linear or circular polarization.

The possibility of *imperfect* quantum cloning has recently attracted considerable attention in connection with quantum information studies. In the present context it is worth noting that Gisin [10] has obtained a bound on the fidelity of quantum cloning by imposing the requirement of causality, and finds that the maximum allowed fidelity is equal to the optimal quantum cloning fidelity obtained by Bužek and Hillery [11]. He notes that, 'Once again, quantum mechanics is right at the border line of contradicting relativity, but does not cross it'.

2.2.3. Remarks. Another interesting idea is a 'superluminal quantum Morse telegraph' involving EPR polarization-correlated photon pairs and a Michelson interferometer in which one of the mirrors is replaced by a phase-conjugating mirror (PCM) [12]. In contrast to an ordinary mirror, where circular (but not linear) polarization is reversed by reflection, polarization does not change upon 'reflection' from a PCM. It would seem therefore that, when one of the ordinary mirrors of a Michelson interferometer is replaced by a PCM, there will be interference of the two propagation paths when the incident light is linearly polarized but not when it is circularly polarized. (In the latter case the fields from the PCM and the ordinary mirror are orthogonally polarized and so do not interfere.) With a steady stream of polarization-correlated EPR pairs, therefore, an observer A would presumably be able to superluminally communicate to an observer B, equipped with the PCM-modified Michelson, whether he is measuring linear or circular polarization.

The interference properties assumed for the PCM-modified Michelson interferometer have in fact been verified experimentally for coherent laser fields. However, a simple calculation shows that for a single incident photon there is no interference between the two paths, regardless of the photon's polarization [13]. This is related to the fact that, like an amplifier, a PCM must have quantum noise—it can spontaneously emit a photon whose polarization is uncorrelated with that of the incident photon [14].

It must be impossible to have an apparatus that could *locally* determine the polarization state of a photon, without making measurements on a (nonlocal) EPR partner. (More generally, it must be impossible to locally determine the eigenstate of a single particle.) Similarly it must be impossible to have an apparatus that faithfully preserves the polarization of a photon entering with (arbitrary) linear polarization while reversing the helicity of a photon entering with circular polarization [15]. Otherwise, as examples such as these suggest, superluminal communication would be possible.

2.3. Fermi's problem and photodetection theory

Another 'causality problem' that has received much attention is a calculation presented by Fermi in 1932 [16]. Fermi wanted to show, with a simple model, how quantum electrodynamics accounts for the propagation of light from a source to a detector. In his model there are two two-state atoms separated by a distance r in free space, in a state at $t = 0$ in which atom A is in its upper-energy state, atom B is in its lower state, and there are no photons. Fermi wanted to show that the probability for atom B to be excited remains zero until the time $t = r/c$ it takes for light to propagate from A to B. However, he made an approximation that, as a matter of principle, rendered his conclusion invalid, although his result for the probability was correct [17].

In the model considered by Fermi each component of the electric field operator associated with either atom in the Heisenberg picture has the form [17]

$$\hat{E}(\boldsymbol{r}, t) \propto \int_0^t \mathrm{d}t' \, \hat{\sigma}_x(t') \int_{-\infty}^{\infty} \mathrm{d}\omega \, \omega^2 \mathrm{e}^{\mathrm{i}\omega(t'-t+r/c)} \propto \theta(t - r/c) \frac{\partial^2}{\partial t^2} \hat{\sigma}_x(t - r/c), \tag{6}$$

where for simplicity we consider only the far field. $\hat{\sigma}_x$ is the two-state Pauli operator in the standard notation, and $\theta(t)$ is the unit step function. The field operator has the same form as its classical counterpart, and in particular it depends on atom operators at the retarded time $t - r/c$ and vanishes for $t < r/c$. Atom B therefore cannot be excited before $t = r/c$.

Fermi, however, restricted himself at the outset to energy-conserving 'essential states', and his neglect of other states is equivalent to a rotating-wave approximation (RWA). When formulated in the Heisenberg picture, the RWA implies the effective electric field operator $\hat{E}(\boldsymbol{r}, t) = \hat{E}^{(+)}(\boldsymbol{r}, t) + \hat{E}^{(-)}(\boldsymbol{r}, t)$, where $\hat{E}^{(-)}(\boldsymbol{r}, t) = \hat{E}^{(+)}(\boldsymbol{r}, t)^\dagger$ and

$$\hat{E}^{(+)}(\boldsymbol{r}, t) \propto \int_0^t \mathrm{d}t' \, \hat{\sigma}(t') \int_0^{\infty} \mathrm{d}\omega \, \omega^2 \mathrm{e}^{\mathrm{i}\omega(t'-t+r/c)}, \tag{7}$$

where $\hat{\sigma}$ is the two-state lowering operator. In contrast to the exact expression for the electric field operator (equation (6)), *the positive- and negative-frequency parts of the field are not retarded.* Fermi circumvented the problem by extending the lower limit of integration over ω to $-\infty$, in which case $\hat{E}^{(+)}(\boldsymbol{r}, t) \propto \theta(t - r/c)(\partial^2/\partial t^2)\hat{\sigma}(t - r/c)$ and it follows that atom B cannot be excited before the time $t = r/c$. His extension of the integration range is a good approximation, and it leads to the correct probability for atom B to be excited [17]. But by making this approximation Fermi did not rigorously prove that atom B cannot be excited before $t = r/c$.

A very similar situation occurs in the standard theory of photodetection. There one assumes (energy-conserving) detection processes in which electrons gain energy as photons are annihilated. This RWA leads to field correlation functions involving normally ordered positive- and negative-frequency field operators which, as already noted, are not retarded. For this reason the theory has been criticized for being 'acausal'. However, causality is easily restored, as in Fermi's problem, by working with the *full* (causal) electric field operator $\hat{E}(\boldsymbol{r}, t)$ and making the RWA at a later stage of the formulation rather than at the beginning [17].

Fermi's problem has been the subject of a rather large literature, some of which is cited in [17].

3. Causality in electromagnetic wave propagation

The causal relation $\boldsymbol{P}(\omega) = \epsilon_0[\kappa(\omega) - 1]\boldsymbol{E}(\omega)$ between the polarization $\boldsymbol{P}$ and the electric field $\boldsymbol{E}$ at frequency ω has the input–output form discussed in section 2.1, and it follows that the dielectric constant $\kappa(\omega)$ has an analytic continuation in the upper half of the complex ω plane. Aside from the possibility of branch points [2], it follows that the refractive index $n(\omega) = \sqrt{\kappa(\omega)}$ has the same property.

Consider the propagation of a scalar plane wave incident from vacuum onto a dielectric medium occupying the half-space $z > 0$. Let the incident field be $E(0, t) =$

$\int_{-\infty}^{\infty} \mathrm{d}\omega\, A(\omega)\mathrm{e}^{-\mathrm{i}\omega t}$. Propagation over the distance z multiplies each Fourier component of the field by $\exp(\mathrm{i}kz) = \exp[\mathrm{i}\omega n(\omega)z/c]$, so that $E(z, t) = \int_{-\infty}^{\infty} \mathrm{d}\omega\, A(\omega)\mathrm{e}^{-\mathrm{i}\omega[t-n(\omega)z/c]}$ or

$$E(z, t) = \int_{-\infty}^{\infty} \mathrm{d}t'\, G(z, t - t')E(0, t'), \tag{8}$$

where

$$G(z, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega[\tau-n(\omega)z/c]}. \tag{9}$$

This equation should, of course, be modified to include the Fresnel transmission coefficient at the interface at $z = 0$, but this does not affect any fundamental results of interest here.

These simple expressions describe a rich variety of propagation effects. Sommerfeld and Brillouin [18] assumed a particular form for the complex refractive index (they were concerned with regions of anomalous dispersion) and showed that signal velocities must be less than c in a medium with such a refractive index. Kramers (1927) showed that this is true in any medium satisfying the dispersion relation (section 2.1), and later Krönig (1942) proved that the dispersion relation is a necessary and sufficient condition for causality (see [1] for references and discussion).

We shall discuss in a bit more detail the classic work of Sommerfeld and Brillouin, but let us first review the important result, which is independent of the detailed form of $n(\omega)$, that the signal velocity cannot exceed c. If $n(\omega)$ is analytic in the upper half of the complex ω plane, *and if* $n(\omega) \to 1$ as $\omega \to \infty$, then it follows from equation (9) and a simple application of Cauchy's theorem that $G(z, \tau) = 0$ for $\tau < z/c$. It follows that if the incident field $E(0, t) = 0$ for $t < 0$, then $E(z, t) = 0$ for $t < z/c$. Sommerfeld and Brillouin defined a *signal* as a train of oscillations that starts from zero at some instant. Then, according to this definition, *no signal can propagate faster than c*. In fact the wave*front* that begins at $z = t = 0$ propagates at exactly the velocity c.

We wish to emphasize the two crucial assumptions leading to this conclusion: (1) the induced polarization is causally related to the electric field that produces it, so that the refractive index must be analytic in the upper half of the complex plane[2]; (2) the refractive index approaches unity as the frequency approaches infinity.

It is, of course, physically reasonable that $n(\omega) \to 1$ as $\omega \to \infty$, and furthermore the refractive index for virtually any system one imagines has an analytic continuation in the upper half-plane[3]. Consider, for instance, a dilute monatomic gas of N atoms per unit volume, such that the transition from the ground level 1 to the first excited level 2 makes the dominant contribution to the refractive index at frequency ω. Then

$$n(\omega) \approx 1 + \frac{Ne^2 f}{2m\epsilon_0} \frac{1}{\omega_0^2 - \omega^2 - 2\mathrm{i}\gamma\omega}, \tag{10}$$

where ω_0 is the $1 \leftrightarrow 2$ transition frequency, f is the oscillator strength for the transition, and γ is a damping rate, proportional to the homogeneous linewidth. $n(\omega)$ here is analytic in the upper half-plane and behaves asymptotically as $1 + \mathrm{O}(\omega^{-2})$. In this and similar models for the propagation of light in dielectrics, the field $E(z, t)$ is zero at position z for all times $t < z/c$ if $E(0, t)$ is zero at $t = 0$.

[2] If we use $\exp(\mathrm{i}\omega t)$ instead of $\exp(-\mathrm{i}\omega t)$ for the time dependence of the frequency component ω of the field, we would be led to demand from causality that $n(\omega)$ should be analytic in the *lower* half of the complex plane.

[3] An exception is the case where the damping mechanism is radiative reaction as described by the classical Abraham–Lorentz theory. $n(\omega)$ in the upper half-plane in this case has a pole resulting from an ω^3 appearing in a denominator. In fact this model is well known to be acausal. But the famous 'preacceleration' problem here occurs on such a short timescale that quantum effects must be considered; i.e., the classical model breaks down.

What happens after $t = z/c$ depends on the particular form of $n(\omega)$ and $E(0, t)$. Because the medium cannot respond instantaneously to the applied field, the wave*front* velocity is always equal to c. After $t = z/c$ there are 'precursors' to the main part of the propagating field, the first precursor arising from the high-frequency parts of the field. A detailed discussion of precursors is given by Oughstun and Sherman [19].

The condition $n(\infty) \to 1$, and the related property that the susceptibility goes to zero as $\omega \to \infty$, can be deduced fairly generally for electrons interacting with the electromagnetic field in the dipole approximation [20], and therefore for the case of greatest practical interest for the propagation of light in dielectrics. It might be noted, however, that there are various situations in which $n(\infty) = 1$ has not been rigorously established [21], and in such cases there is no proof, aside from general considerations of causality, that the front velocity, defined in general as $c/\mathrm{Re}[n(\infty)]$, must be c, or that the signal velocity cannot exceed c.

4. Group velocity

Writing $E(z, t) = \mathcal{E}(z, t)\exp[-\mathrm{i}(\omega_L t - k_L z)]$, where $k_L = k(\omega_L)$, and differentiating (8) with respect to z, we obtain

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}t'\,\mathcal{E}(0, t')\int_{-\infty}^{\infty} \mathrm{d}\omega\,[k(\omega_L + \omega) - k_L]\mathrm{e}^{-\mathrm{i}\omega(t-t')}\mathrm{e}^{\mathrm{i}[k(\omega_L+\omega)-k_L]z}. \tag{11}$$

The Taylor expansion

$$k(\omega_L + \omega) = k_L + \left(\frac{\mathrm{d}k}{\mathrm{d}\omega}\right)_{\omega_L}\omega + \frac{1}{2}\left(\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}\right)_{\omega_L}\omega^2 + \cdots \tag{12}$$

in equation (11) gives

$$\frac{\partial \mathcal{E}}{\partial z} + \left(\frac{\mathrm{d}k}{\mathrm{d}\omega}\right)_{\omega_L}\frac{\partial \mathcal{E}}{\partial t} + \frac{\mathrm{i}}{2}\left(\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}\right)_{\omega_L}\frac{\partial^2 \mathcal{E}}{\partial t^2} + \cdots = 0. \tag{13}$$

It often happens that $(\mathrm{d}^2k/\mathrm{d}\omega^2)_{\omega_L}\partial^2\mathcal{E}/\partial t^2$ and all the other terms involving higher derivatives of $\mathcal{E}$ in equation (13) are negligible compared with the first-derivative terms. If, furthermore, absorption at frequency ω_L is sufficiently weak that $(\mathrm{d}k/\mathrm{d}\omega)_{\omega_L}$ may be taken to be real, then

$$\frac{\partial \mathcal{E}}{\partial z} + \frac{1}{v_g}\frac{\partial \mathcal{E}}{\partial t} = 0, \tag{14}$$

where the group velocity

$$v_g = \left(\frac{\mathrm{d}\omega}{\mathrm{d}k}\right)_{\omega_L} = \frac{c}{(n_R + \omega\,\mathrm{d}n_R/\mathrm{d}\omega)_{\omega_L}}, \tag{15}$$

n_R being the real part of the refractive index. In this approximation a pulse envelope propagates without change of shape or amplitude at the group velocity:

$$E(z, t) = \mathrm{e}^{-\mathrm{i}(\omega_L t - k_L z)}\mathcal{E}(0, t - z/v_g). \tag{16}$$

In a spectral region of 'normal' dispersion ($(\mathrm{d}n_R/\mathrm{d}\omega)_{\omega_L} > 0$), the group velocity is less than the phase velocity (c/n_R). Because v_g can exceed c in a region of 'anomalous' dispersion ($(\mathrm{d}n/\mathrm{d}\omega)_{\omega_L} < 0$), and because v_g was generally thought to be the velocity of energy propagation, $v_g > c$ was at one time thought to be in conflict with the special theory of relativity. This conflict was resolved when Sommerfeld and Brillouin proved that the *signal* velocity cannot exceed c even in a region of anomalous dispersion (section 3).

However, confusion surrounding the meaning of group velocity did not end with the work of Sommerfeld and Brillouin. Even today, and even in well known textbooks and monographs, one finds the assertion that the concept of group velocity lacks physical significance when $v_g > c$ (or has other interesting values, as discussed later) because, in contradistinction to equation (16), the pulse will become highly distorted as a consequence of strong dispersion implied by $v_g > c$. This is a sensible but incorrect conclusion in general, as we shall see. Worse, it is often implied that there would be a conflict with special relativity if the group velocity were larger than c. This is false because, to repeat, group velocity is not in general a signal velocity. The notion of group velocity represents an *approximation* to the actual state of affairs in which, as we have seen, signals cannot propagate faster than c.

To cite just one example where the group velocity has been implicitly assumed to be a signal velocity, consider the propagation of x-rays in glass, in which case we can take $\omega \gg \omega_0$ and ignore the damping term in equation (10): $n(\omega) \approx 1 - a/\omega^2$, with $a > 0$. Then the phase velocity $c/n(\omega_L) > c$ while the group velocity $v_g = c/[1 + a/\omega^2] < c$. Thus it has been concluded that, 'although the phases can travel faster than the speed of light, the modulation signals travel slower, and that is the resolution of the apparent paradox!' [22]. The conclusion is of course correct, but only because in this example the group velocity does turn out to be the same as the (modulation) signal velocity for practical purposes, because x-rays in glass are far from any absorption resonances. In a different medium, where a could be *negative* and the group velocity greater than c, the same argument would lead to a serious 'paradox' indeed.

5. Superluminal group velocities

Close to a resonance the refractive index (10) can be approximated by

$$n(\omega) = 1 + \frac{K}{\omega_0 - \omega - \mathrm{i}\gamma} = n_R(\omega) + \mathrm{i}n_I(\omega), \tag{17}$$

where $K = Ne^2 f/4m\omega_0\epsilon_0$ and

$$n_R(\omega) = 1 + K\frac{\omega_0 - \omega}{(\omega_0 - \omega)^2 + \gamma^2}, \tag{18}$$

$$n_I(\omega) = K\frac{\gamma}{(\omega_0 - \omega)^2 + \gamma^2}. \tag{19}$$

$n_R(\omega)$ decreases with increasing frequency near an absorption line (figure 1). Such anomalous dispersion was observed by Wood in 1904 (see, for instance, [23]) and implies (equation (15)) that the group velocity can exceed c.

Our discussion thus far has implicitly assumed an absorbing medium in which the number density N is approximately the density of atoms in the ground state. More generally N should be replaced by the difference of ground-state and excited-state populations for the transition of frequency ω_0. Thus, in the case of an *amplifying* medium, where this difference is negative, K is negative and we have normal dispersion (and $v_g < c$) close to the resonance frequency ω_0. The curve of $n_R(\omega) - 1$ versus ω in this case is simply reversed in sign compared with the curve of figure 1, so that the dispersion becomes anomalous when the field frequency is tuned *away* from ω_0. Thus, depending on the detuning $\omega - \omega_0$ in the vicinity of a resonance, the group velocity can exceed c in amplifiers as well as absorbers.

More generally, Bolda *et al* [24] have shown that for any dispersive dielectric medium there must be a frequency for which the group velocity is 'abnormal', i.e. larger than c, infinite or negative. In an absorber (amplifier) the group velocity is 'abnormal' (normal) at the frequency at which the absorption (amplification) is greatest.

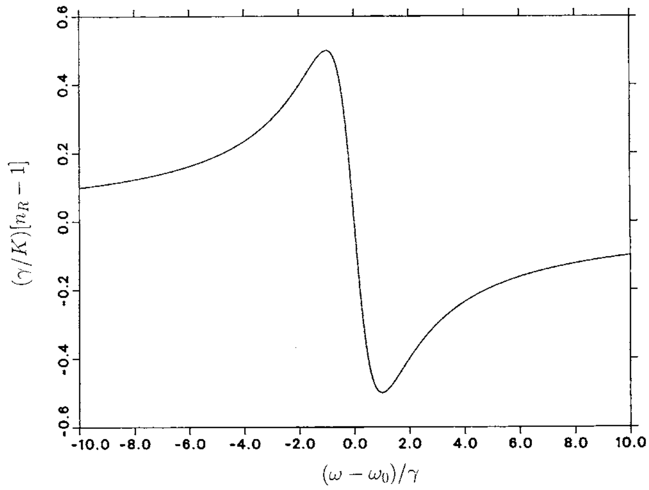

Figure 1. $(\gamma/K)[n_R(\omega) - 1]$ (equation (18)) versus $(\omega - \omega_0)/\gamma$.

Basov *et al* [25] demonstrated that the group velocity of a laser pulse in an amplifier could exceed c due to a pulse reshaping in which the front part of the pulse leaves less gain available for the back part, resulting in an advancement of the peak of the pulse. Icsevgi and Lamb [26] argued that Basov *et al* in their analysis assumed 'an unphysical input pulse extending to infinity at both ends'. The measurement of pulse velocities greater than c does not contradict special relativity because 'an experimental apparatus can only trace the bulk of the pulse', which does not constitute a signal. The point made by Icsevgi and Lamb touches on the crucial question of what defines a signal; we return to this point later.

Garrett and McCumber [27] showed that superluminal group velocities can appear when a Gaussian pulse propagates in an absorbing medium, provided that the pulse bandwidth is much smaller than the width of the absorption line and the medium is sufficiently short. This results from a pulse reshaping and advancement process in which the back part of the pulse is more strongly absorbed than the front part. Equation (15) indicates that the group velocity can be infinite or even negative if anomalous dispersion is sufficiently strong ($\mathrm{d}n_R/\mathrm{d}\omega$ sufficiently negative). An infinite group velocity means that the peak of the pulse emerging from the medium occurs at the same instant as the peak of the pulse entering the medium. A negative group velocity means that the peak of the emerging pulse occurs at an earlier time than the peak of the incident pulse. Furthermore this can occur without significant pulse distortion, albeit with significant absorption. Experimental observations of these remarkable effects were reported in 1982 [28].

5.1. Photon tunnelling experiments

More recent interest in 'abnormal' group velocities is due in considerable part to the experimental and theoretical work of Chiao's group, which has been reviewed previously [29–31]. This work, *inter alia*, answered some important and long-standing questions about tunnelling times.

MacColl [32] in 1932 considered the transmission and reflection of a wavepacket incident on the potential barrier defined by $V(x) = 0$ for $x < 0$ and $x > a$, and $V(x) = V_0 > 0$ for $0 < x < a$. It is important to note that he chose an initial wavepacket that did not vanish, but was very small, for $x > a$; this initial wavepacket, as opposed to one that identically vanishes for $x > a$, was chosen so that none of the energies E making up the incident packet exceed V_0. He found that 'the transmitted packet appears at the point $x = a$ at about the time at which the incident packet reaches the point $x = 0$, so that there is no appreciable delay in the transmission of the packet through the barrier'. Defining the tunnelling time in terms of how long it takes for the peak of the wavepacket at $x = a$ to occur relative to the peak of the incident packet at $x = 0$, one would conclude that the tunnelling is superluminal. But note that MacColl's assumption for the initial wavepacket means there is no sharp front and no *signal* in the sense of Sommerfeld and Brillouin. That is, there is no violation of Einstein causality implied by MacColl's zero delay, even though the peak of the wavepacket appears to cross the barrier superluminally. The velocity of the peak of the wavepacket for the particle is analogous to the group velocity of an electromagnetic pulse.

It is well known that evanescent waves in optics are analogous to tunnelling wavefunctions in wave mechanics. The Helmholtz equation $\nabla^2\mathcal{E} + (n^2\omega^2/c^2)\mathcal{E} = 0$ for the amplitude of a scalar monochromatic wave has the same form as the time-independent Schrödinger equation, $\nabla^2\psi + (2m/\hbar^2)(E - V)\psi$, making the occurrence of an evanescent optical wave (imaginary n) analogous to particle tunnelling ($E < V$).

Chiao *et al* performed experiments in which the central frequency of a single-photon wavepacket was that for minimum transmission through a multilayered dielectric consisting of alternating layers of high and low n. The exponentially decaying, evanescent behaviour of the transmitted wave is analogous to quantum tunnelling—in fact the situation is analogous to the Kronig–Penney model for the propagation of electrons in a crystal. Near the frequency of minimal transmission in the experiments, the group velocity approximation is quite accurate, and therefore the tunnelling wavepacket should suffer little distortion, although the transmission is, of course, very small.

Spontaneous parametric down-conversion was used to simultaneously generate pairs of photons at a wavelength of 702 nm. Two pinholes used to select the photon pairs determine, by their size and the phase-matching condition, the bandwidth of the down-converted light (see, for example, [33] for a simplified treatment). In the experiments the resulting photon wavepackets had a bandwidth of $\sim$6 nm and a temporal width $\sim$20 fs. Using coincidence photon counting and an application of a Hong–Ou–Mandel interferometer [34] to measure the femtosecond-scale delays between photons that traversed the tunnel barrier and their twins that passed through air, Chiao *et al* were able to determine the photon tunnelling times. Figure 2 shows photon coincidence rate data versus the path delays with and without the tunnel barrier in place. The negative delay found with the barrier means that a single photon tends to pass through the barrier faster than it would propagate through an equal distance in air. Effective tunnelling velocities of about $1.7c$ were inferred from the measured photon coincidence rates.

These experiments demonstrated that the tunnelling process can indeed be 'superluminal', as predicted by MacColl [32] and elaborated upon by Wigner [35], Hartman [36] and others. We refer the reader to earlier reviews [29–31] for discussions about different definitions and theories of tunnelling times, as well as to papers by Büttiker and Thomas (see [37] and references therein).

5.2. Gain-doublet experiments

In the experiments of Chiao *et al* the tunnelling probability is very small; similarly, in the experiments of Chu and Wong [28], there was substantial attentuation of the pulses propagating in the absorbing medium. The observation of distortionless pulses propagating with superluminal group velocity *and* relatively small change in amplitude has recently been reported by Wang *et al* [38]. In these experiments a gain doublet is employed, such that in the

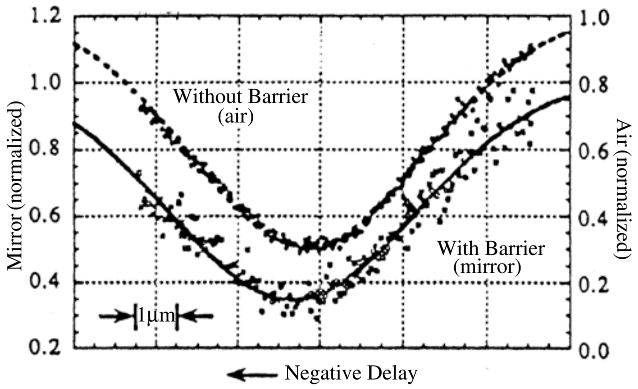

Figure 2. Data of Chiao *et al* for the coincidence rate versus the path difference of twin photons produced by spontaneous parametric downconversion, one photon passing through a tunnel barrier and the other through a column of air of the same length as the barrier. From [31], with permission.

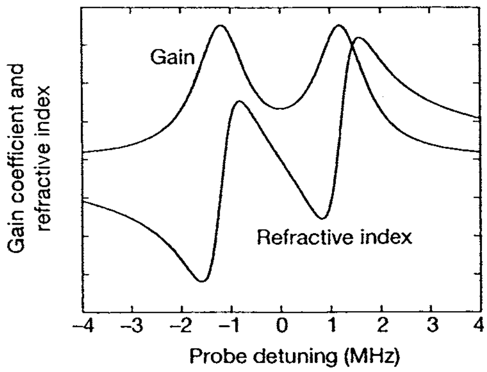

Figure 3. Gain and refractive index in the vicinity of a gain doublet. From [38], with permission.

spectral region between two gain peaks there is strong anomalous dispersion but little gain (or absorption) [39] (figure 3). In such a spectral region the group velocity can differ significantly from c while the pulse suffers little change in either amplitude or shape.

Wang *et al* used a 6 cm caesium cell coated with paraffin, which allows atoms to maintain their spin polarization when they collide with the walls. They prepared the caesium atoms in the three-state system shown in figure 4 by optical pumping with polarized light. Two right-hand circularly polarized, continuous-wave Raman pump beams, shifted in frequency by 2.7 MHz, are incident on the cell; their electric field amplitudes E_1 and E_2 are indicated in figure 4. The state $|2\rangle$ in figure 4 is the final state of the Raman transition, while the state $|0\rangle$ serves as the primary intermediate state. A continuous-wave probe field (E_p in figure 4) was varied in frequency with an acousto-optical modulator and used to measure the gain and refractive index as a function of frequency; results conforming accurately to the curves in figure 3 were obtained. The predicted group velocity was $v_g = -c/330$. Then a weak, nearly Gaussian probe pulse (3.7 μs FWHM) was used to measure transmitted pulse intensity profiles and propagation times. It was verified that the peak of the transmitted pulse appears at the end of the cell before the peak of the incident pulse appears at the entrance to the cell, consistent with the prediction of a negative group velocity. The transmitted peak occurred 62 ns before the incident peak. Thus the transmitted peak goes about 18 m from the cell before the incident

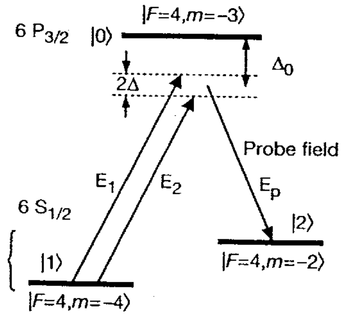

Figure 4. Approximate level diagram of caesium in the experiments of Wang *et al*. From [38], with permission.

peak even arrives. Since the time for light travelling at c to traverse the 6 cm cell is about 0.2 ns, the 62 ns advance implies a group velocity in the cell of about $-c/310$, consistent with the measured refractive index and the formula (15) for the group velocity.

Note that the 62 ns advance of the pulse peak is a small fraction of the pulse width. A similar remark applies to the data of Chiao *et al* (figure 2). As far as the author is aware the pulse advance is small compared with the pulse width in all 'superluminal' experiments reported thus far. We shall return to this point in section 7.

It may be worthwhile to elaborate slightly on the concept of negative group velocity. Consider a medium of refractive index $n(\omega)$ such that $v_g < 0$ at the central frequency ω_L of an incident pulse. Let the medium occupy the region from $z = 0$ to $z = L$, outside which $n(\omega) = 1$. For $z < 0$ we have $E(z,t) = E(0, t - z/c) \equiv f(t - z/c)$. For $0 < z < L$ equations (8) and (9) give

$$E(z,t) \approx \mathrm{e}^{\mathrm{i}\omega_L n(\omega_L) z/c} \mathrm{e}^{-\mathrm{i}\omega_L z/v_g} f(t - z/v_g) \tag{20}$$

in the group velocity approximation ($\omega n(\omega) \approx \omega_L n(\omega_L) + (c/v_g)(\omega - \omega_L)$ in equations (8) and (9)). For $z > L$ the field propagates in free space after having incurred a phase shift $[n(\omega) - 1]\omega L/c$ in each frequency component. Then, in the notation of section 3,

$$\begin{aligned} E(z,t) &= \int_{-\infty}^{\infty} \mathrm{d}\omega\, A(\omega) \mathrm{e}^{\mathrm{i}[n(\omega)-1]\omega L/c} \mathrm{e}^{-\mathrm{i}\omega(t - z/c)} \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t'\, E(0,t') \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{\mathrm{i}\omega(t'-t)} \mathrm{e}^{\mathrm{i}\omega z/c} \mathrm{e}^{-\mathrm{i}\omega L/c} \mathrm{e}^{\mathrm{i}\omega n(\omega) L/c} \\ &\approx \mathrm{e}^{\mathrm{i}\omega_L n(\omega_L) L/c} \mathrm{e}^{-\mathrm{i}\omega_L L/v_g} f(t - z/c + L/c - L/v_g) \end{aligned} \tag{21}$$

in the group velocity approximation. In figure 6 we plot $|E(z,t)|^2$ at different times, assuming a Gaussian pulse, $|f(t - z/c)|^2 = \exp[-(t - z)^2]$, with $c = 1$, $L = 10$ and $v_g = -c/1.5$. The pulse in the medium ($v_g < 0$) propagates from $z = L$ to $z = 0$, where in a sense it 'annihilates' the incident pulse, after which all that remains is the exit pulse propagating in free space for $z > L$. The peak of the exit pulse is seen to begin before the peak of the incident pulse reaches $z = 0$.

5.3. Other experiments and viewpoints

As noted earlier, experimental observations of group velocities $> c$ go back a long way. Chiao and Steinberg [30] have masterfully reviewed the relevant work up to about 1996, and there is no need to discuss this work in any detail here, although a few remarks are appropriate regarding the *interpretation* of earlier experiments as well as more recent ones.

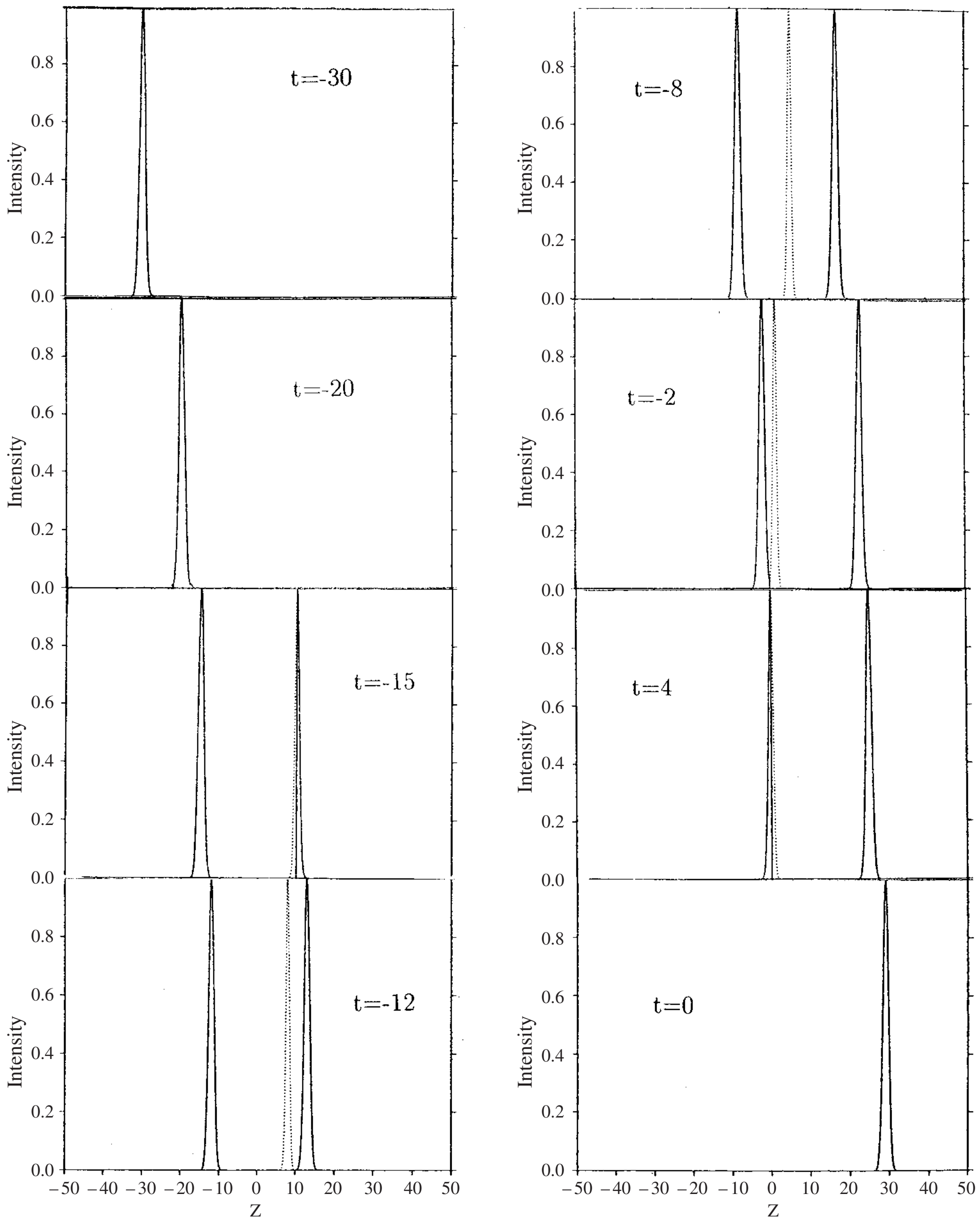

Figure 5. The intensity $|E(z,t)|^2$ at different instants of time for an incident Gaussian pulse incident on a medium with negative group velocity. The medium is assumed to occupy the region $0 < z < 10$, and units are chosen such that $c = 1$, $v_g = -c/1.5$, and the incident pulse intensity is $\exp[-(t-z)^2]$. The dashed curve is the intensity inside the medium.

A series of elegant experiments by Nimtz and others (see [40] and references therein) has demonstrated evanescent-wave superluminal group velocities in microwave waveguides. To see how this can come about, consider the TE_{01} mode of a waveguide of width a in which the refractive index is n except for a rectangular air gap ($n \approx 1$) [41]. The dispersion relation is then $\omega_c^2 + K_z^2c^2 = \omega^2/c^2$ in the gap and $\omega_c^2 + k_z^2c^2 = n^2\omega^2/c^2$ elsewhere, with $\omega_c = \pi c/a$.

Thus ω can be chosen such that K_z is imaginary while k_z is real, and in this case the propagation in the waveguide is analogous to tunnelling, and the evanescent wave can cross the gap with a superluminal group velocity.

In one experiment Nimtz *et al* encoded Mozart's 40th symphony on a microwave and reported that this 'signal' was transmitted at $4.7c$! As discussed below, the superluminal group velocity of the transmitted wave form does not violate Einstein causality because it does not represent a superluminal transmission of *information*. In particular, as noted by Nimtz *et al*, there is no superluminal transmission here of a sharp wave*front*. Whether an actual 'signal' is transmitted becomes partly a question of semantics, but, according to the definition of a signal as a carrier of new information, there is no superluminal signal propagation in these experiments, and therefore no violation of Einstein causality. As discussed in the following section, and as remarked by Chiao and Steinberg [30], '[the] appearance of a wave form faster than c is in itself nothing surprising'.

Suppose there is destructive interference between a wave $\psi(t)$ and a retarded and attenuated portion $\eta\psi(t-\Delta t)$. The superposition of these waves in the first-order approximation is $\psi(t)-\eta\psi(t-\Delta t)\approx(1-\eta)\psi(t+\chi\Delta t)$, where $\chi=\eta/(1-\eta)>0$; the destructive interference therefore provides an extrapolation of ψ from t to $t+\chi\Delta t$. This simple observation is relevant to the tunnelling experiments: Chiao and Steinberg [30] discuss the fact that 'the interference at work in tunnelling has the effect of advancing the incident wave form due to the first derivative term of Taylor's theorem', and that this advancement occurs 'without any need for *information* about the later behaviour of the incident field'. Chiao and Steinberg [30] observe that

> The time advance being discussed is well under 1 ns in Nimtz's experiments. An acoustic wave form, on the other hand, has a useful bandwidth on the order of 20 kHz, which is to say that no significant deviation from a low-order Taylor expansion occurs in less than about 50 μs. To predict where the wave form would be 50 μs in advance requires little more than a good eye; to predict it 1 ns in advance hardly even requires a steady hand.

Nimtz *et al* [40,42] have argued that their evanescent fields do not satisfy Einstein causality because, as a practical matter, a 'signal' is limited in its frequency spread, or in other words a real-world signal cannot have a sharp turn-on or turn-off. Obviously the disagreement over whether a signal can or cannot propagate faster that c hinges on the definition of a signal. The writer adheres to the conventional view—which is not always made explicit in discussions of Einstein causality—that a signal is something that conveys information, and as such must involve a *discontinuity* in a waveform or one of its derivatives. A signal defined as such does not violate Einstein causality. This is the viewpoint advocated many years ago by Icsevgi and Lamb [26] in their criticism of the work of Basov *et al* [25]. On the other hand the arguments of Nimtz *et al* raise the valid point that the term 'signal' needs to be better defined, especially when imperfect detectors and quantum effects are considered.

5.4. No violation of Einstein causality

For the purpose of explaining why none of these intriguing experimental results do not violate the principle of Einstein causality—the principle that no signal can propagate faster than c—it is useful to cast the group velocity approximation discussed in section 4 in a different form. Assume $E(0,t)=A(t)\exp(-\mathrm{i}\omega_L t)$, where $A(t)$ varies sufficiently slowly on timescales $\sim\omega_L^{-1}$ that we can replace $k(\omega)=n(\omega)\omega/c$ in equations (8) and (9) by $k(\omega_L)+(\mathrm{d}k/\mathrm{d}\omega)_{\omega_L}(\omega-\omega_L)=k_L+(\omega-\omega_L)/v_g$. It is straightforward to show from equations (8) and (9) that in this approximation [43]

$$E(z,t)=\mathrm{e}^{-\mathrm{i}(\omega_L t-k_L z)}\mathrm{e}^{[c^{-1}-v_g^{-1}]z\partial/\partial t}A(t-z/c)=\mathrm{e}^{-\mathrm{i}(\omega_L t-k_L z)}A(t-z/v_g). \quad (22)$$

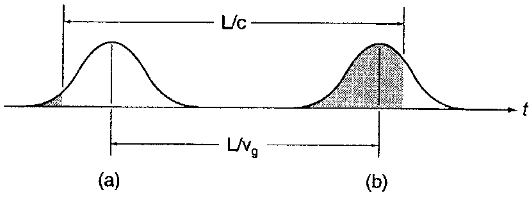

Figure 6. Incident (a) and transmitted (b) pulses for a propagation length z and group velocity $v_g > c$. The shaded portion of (b) is completely determined by the shaded portion of (a).

Thus, in the group velocity approximation, propagation over a distance z corresponds to an analytic continuation over the time $z/c - z/v_g$ of the vacuum-propagated pulse envelope $A(t-z/c)$. Therefore a superluminal group velocity does not imply a superluminal propagation of information, since there is no information in $A(t - z/v_g)$ that is not already contained in $A(t - z/c)$.

New information is propagated only if $A(t - z/c)$ does not have an analytic continuation. In this case the second equality in (22) is invalid, while the first equality holds up to a time at which $A(t - z/c)$ or one of its derivatives has a discontinuity. Thereafter the pulse evolution becomes much more complicated than a simple undistorted propagation at the superluminal group velocity. The point of singularity behaves like a Sommerfeld–Brillouin front, which propagates at c. In other words, a true signal evidently requires nonanalyticity, and cannot transmit information at a velocity $> c$.

The case of a differentiable (e.g., Gaussian) waveform propagating without distortion at a superluminal group velocity is nevertheless remarkable when one considers that (1) points at time t on the transmitted pulse are causally determined by points at $t < z/c$ on the incident pulse, and yet (2) the transmitted pulse advances at the velocity $v_g > c$. This means, as indicated in figure 5, that if $(z/c - z/v_g)$ is much larger than the pulse duration, the peak of the transmitted signal is reconstructed entirely from a small tail of the incident pulse; if $(z/c - z/v_g)$ is large enough, nearly the entire transmitted pulse is reconstructed by analytic continuation of a tiny portion of the incident pulse.

Note that the peak of the transmitted pulse is not causally connected to the peak of the incident pulse, so that in particular the observation that the pulse peak moves superluminally does not contradict Einstein causality. A simple analogy would be the motion of a spot of light made by shining a rotating flashlight onto a distant wall. The spot can in principle move superluminally, but there is no violation of causality because the spot at one instant is not the source of the spot at a later instant. In this same sense the experimental observations of pulse peaks propagating faster than c do not contradict Einstein causality.

The recent experimental observations of superluminal group velocities involve a linear response of the medium to the field, and the exchange of energy between the medium and the front and back parts of the pulse, leading to the pulse advancement, can be interpreted using classical spectral arguments [44].

Linearity also makes it straightforward to perform a quantum mechanical calculation using a simple model in which an absorbing dielectric is described as a collection of identical two-level atoms and the source of light is a single excited atom outside the dielectric [45]. It is shown that, if the source atom of transition frequency ω_0 is suddenly excited at time $t = 0$, then the probability of detecting a photon at a point inside the dielectric and at a distance z from the source atom is zero before the time z/c. This is the analogue of the classical result that a sharp wavefront cannot propagate faster than c. However, if the probability $P(t)$ that the source atom is excited varies smoothly in time, then the photon counting rate $R(t)$ at an ideal detector located at z is proportional to $\exp(-2\omega_0 n_I(\omega_0)z/c)P(t - z/v_g)$, where n_I is again the imaginary part of the refractive index. Thus, if $v_g > c$, the peak probability of producing a 'click' at the detector can occur earlier than is possible when there is no medium between the detector and the source atom. This is analogous to what has been observed in the optical tunnelling experiments of Chiao *et al*.

In section 3 we reviewed the Sommerfeld–Brillouin proof that a sharp wavefront, one that jumps discontinuously from zero to a finite value, cannot propagate faster than c. We noted that the front velocity is determined by the infinite-frequency response of the propagation medium (the front velocity is $c/n_R(\infty) = c$). We also noted that Sommerfeld and Brillouin [18] defined a signal as a train of oscillations that starts from zero at some instant, and that according to this definition no signal can propagate faster than c, the front velocity. From the discussion following equation (22) one concludes more generally that the propagation of new information—a signal—requires a discontinuity in a waveform or one of its derivatives [30,43]. The discontinuity involves infinite-frequency components and therefore propagates at c. Chiao and Steinberg [30] define an idealized *signal* as 'the complete set of all the points of nonanalyticity $\{t_0, t_1, t_2, \ldots\}$, together with the values of the input function $f_{\text{in}}(t)$ in a small but finite interval of time inside the domain of analyticity immediately following these points'.

The signal velocity of an optical pulse is sometimes defined as the velocity of propagation of the half-the-peak-intensity point on the leading part of the pulse. Such a 'signal' velocity can exceed c but, as we have seen, it is not really a signal velocity because it does not necessarily convey information that is not already contained in the leading edge of the pulse. Moreover, as noted by Brillouin [18],

> (this) definition of the signal velocity is somewhat arbitrary.... The signal does not arrive suddenly; there is a quick but still continuous transition from the very weak intensity of the (precursors) to that corresponding to the signal. A detector set to detect an intensity equal to 1/4 the final intensity will detect the arrival of the signal in agreement with the above arbitrary definition; if the detector is more or less sensitive, then it will detect the arrival of the signal a little earlier or later.

5.5. *Bessel beams*

The faster-than-c effects described thus far arise from the dispersion of light in a material medium. We now describe some work in which 'superluminal' effects derive solely from the nature of the propagating field. More specifically, we shall describe some work on the propagation of so-called Bessel beams [23,46].

Let us first summarize a few salient features of Bessel beams. Consider a scalar wave of frequency ω propagating in the z direction and having the azimuthally symmetric form

$$E(z,t) = \mathcal{E}(\rho)\mathrm{e}^{-\mathrm{i}(\omega t - k_z z)}, \qquad \rho = \sqrt{x^2 + y^2}. \tag{23}$$

Such a wave is 'diffractionless' in that the intensity $(\propto |\mathcal{E}(\rho)|^2)$ is independent of the propagation distance z. The wave equation $\nabla^2 E - c^{-2}\partial^2 E/\partial t^2 = 0$ implies

$$\frac{\mathrm{d}^2\mathcal{E}}{\mathrm{d}\rho^2} + \frac{1}{\rho}\frac{\mathrm{d}\mathcal{E}}{\mathrm{d}\rho} + (k^2 - k_z^2)\mathcal{E} = 0 \qquad (k = \omega/c), \tag{24}$$

which has the solution

$$\mathcal{E}(\rho) = J_0(k_\rho \rho) \qquad (k^2 = k_z^2 + k_\rho^2) \tag{25}$$

where J_0 is the zeroth-order Bessel function. Writing $k_\rho = k\sin\theta$ and $k_z = k\cos\theta$, and using the integral representation of J_0, we have

$$E(z,t) = J_0(k\rho\sin\theta)\mathrm{e}^{-\mathrm{i}(\omega t - kz\cos\theta)} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{-\mathrm{i}(\omega t - kz\cos\theta)}\mathrm{e}^{\mathrm{i}(kx\sin\theta\cos\phi + ky\sin\theta\sin\phi)}$$
$$= \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi\, \mathrm{e}^{-\mathrm{i}(\omega t - \boldsymbol{q}\cdot\boldsymbol{r})}, \qquad (26)$$

where the wavevector $\boldsymbol{q} = k(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta)$ intersects the z axis at angle θ. Thus a (zero-order) Bessel beam of frequency ω comprises all possible plane waves with wavevectors of magnitude $|\boldsymbol{q}| = \omega/c$ making an angle θ to the z axis. It follows that a Bessel beam can be produced by illuminating a narrow annulus lying in the focal plane of a lens [23, 46].

The phase velocity of the wave (26) is $v_p = c/\cos\theta$. The group velocity along the z direction[4] is $v_g = \partial\omega/\partial k_z = ck_z/k = c\cos\theta$.

The fact that the plane-wave components of the Bessel beam intersect the z axis at the angle θ means that the point of contact with the z axis of all of these waves (or, more precisely, their planes of constant phase) move along the z axis with the velocity $c/\cos\theta$. The z axis therefore appears to 'light up' superluminally. As noted by Saari and Reivelt [47], however, 'This speed is superluminal in a similar way as one gets a faster-than-light movement of a bright stripe on a screen when a plane wave light pulse is falling at the angle θ onto the screen plane'. In other words, the 'superluminal' propagation along z is merely a geometrical effect, analogous to the old 'scissors paradox' in which the point of contact of the two blades could move faster than c while the ends of the blades move with velocity less than c. The points of contact are not causally connected, nor are points of contact along the z axis causally connected in the case of the Bessel beam. There is certainly no violation of Einstein causality, although it has been claimed [48] that the Bessel beam 'superluminality' calls this fundamental principle into question. This claim has been challenged on both theoretical and experimental grounds [49, 50].

In order to propagate a *signal* from, say, $(0, 0, 0)$ to $(0, 0, z)$, information would first have to be sent from $(0, 0, 0)$ to points in the $z = 0$ plane at a distance $\rho = z\tan\theta$ away; this distance gives the location in the $z = 0$ plane of the conical surface on which lie the wavevectors of the plane-wave components that propagate to $(0, 0, z)$ to produce the Bessel beam at $(0, 0, z)$. The information must then propagate over the distance $z/\cos\theta$ from these points to $(0, 0, z)$. The time it takes for the information to be propagated from $(0, 0, 0)$ to $(0, 0, z)$, assuming that the information can be sent at velocity c, is then $t = (z\tan\theta + z/\cos\theta)/c$. Thus the velocity with which information can be propagated from $(0, 0, 0)$ to $(0, 0, z)$ is $z/t = c/(\tan\theta + \sec\theta) \leqslant c$.

6. Propagation of energy

As remarked earlier, the work of Sommerfeld and Brillouin and others was motivated in part by the association at the time of group velocity with the velocity at which electromagnetic energy propagates. The fact that the group velocity can exceed c in a region of anomalous dispersion led to more careful considerations of the meaning of group and signal velocities, as already discussed.

[4] Some authors [47, 48] state that the group velocity is $c/\cos\theta$. The argument for this is evidently that $v_g = \mathrm{d}\omega/\mathrm{d}k_z = (\mathrm{d}\omega/\mathrm{d}k)(\mathrm{d}k/\mathrm{d}k_z) = c/\cos\theta$, i.e. that $k_z = k\cos\theta$ with $\cos\theta$ a constant parameter. The definition of group velocity, however, presumes a wave*packet*, in which case one cannot assume a fixed relation between k and k_z.

In considering the velocity v_E of energy transport it is imperative to recognize the simple fact that part of the energy density is stored for a finite time in the propagation medium. Simple but instructive models based on the Lorentz model of a dielectric lead to the conclusion that v_E, defined as the ratio of the (cycle-averaged) Poynting vector and the total energy density (including that of the medium) is $\leqslant c$ for a monochromatic wave [19, 51]. In the absence of absorption, $v_E = v_g$, and $v_E \approx v_g$ if there is absorption but the field frequency is far from an absorption resonance [51]. Because v_E in this model is based on the assumption of a monochromatic field, it cannot be associated with a signal velocity.

Diener [52] notes that if v_E is defined in the usual way as the Poynting vector divided by the total energy density, it can exceed c and thereby 'be in contradiction with fundamental principles of physics'. He proposes a different definition of the energy transport velocity that gives $v_E \leqslant c$ even if $v_g > c$.

Consider the cycle-averaged electromagnetic energy density u_ω in a medium for which absorption (or amplification) at frequency ω is negligible [53]:

$$u_\omega = \frac{1}{16\pi}\left(\frac{\mathrm{d}}{\mathrm{d}\omega}(\epsilon\omega)|\boldsymbol{E}_\omega|^2 + \frac{\mathrm{d}}{\mathrm{d}\omega}(\mu\omega)|\boldsymbol{H}_\omega|^2\right) \tag{27}$$

in the usual notation. Using $|\boldsymbol{H}_\omega|^2 = (\epsilon/\mu)|\boldsymbol{E}_\omega|^2$ for plane waves, we obtain

$$u_\omega = \frac{n}{8\pi\mu}\frac{\mathrm{d}}{\mathrm{d}\omega}(n\omega)|\boldsymbol{E}_\omega|^2 = \frac{nc}{8\pi\mu v_g}|\boldsymbol{E}_\omega|^2 \qquad (n^2 = \epsilon\mu). \tag{28}$$

Similarly the cycle-averaged Poynting vector $\boldsymbol{S} = (c/4\pi)\boldsymbol{E}\times\boldsymbol{H}$ at frequency ω has magnitude

$$|\boldsymbol{S}_\omega| = \frac{nc}{8\pi c\mu}|\boldsymbol{E}_\omega|^2 = v_g u_\omega, \tag{29}$$

which implies $v_E \equiv |\boldsymbol{S}_\omega|/u_\omega = v_g$.

However, Diener argues that u_ω is the total energy density, including that stored in the medium, so that v_E should *not* be defined as $|\boldsymbol{S}_\omega|/u_\omega$. Consider the Lorentz model of a nonmagnetic medium ($\mu = 1$, $\epsilon = n^2$) in which the electron displacement x satisfies the simple harmonic oscillator equation $\ddot{x} + \omega_0^2 x = (e/m)E_\omega \cos\omega t$. The cycle-averaged electron energy in this model is

$$W_m = \frac{(e^2/4m)E_\omega^2}{(\omega_0^2 - \omega^2)^2}[\omega_0^2 + \omega^2], \tag{30}$$

which can be written, using the Lorentz-model expression

$$\epsilon = 1 + \frac{4\pi N e^2/m}{\omega_0^2 - \omega^2} \tag{31}$$

for N oscillators per unit volume, as

$$N W_m = \frac{1}{16\pi}(\epsilon - 1)E_\omega^2 + \frac{1}{8\pi}n\omega\frac{\mathrm{d}n}{\mathrm{d}\omega}E_\omega^2. \tag{32}$$

This is the energy density stored in the Lorentz-model medium.

The difference

$$u_\omega^{(F)} \equiv u_\omega - N W_m = \frac{1}{16\pi}(\epsilon + 1)E_\omega^2 \tag{33}$$

is then identified by Diener [52] as the 'energy (density) of the electromagnetic field in the proper sense'. He defines the *energy transport velocity* as

$$v_E^{(F)} \equiv |\boldsymbol{S}_\omega|/u_\omega^{(F)}, \tag{34}$$

which, using equations (28), (29) and (33), is found to be

$$v_E^{(F)} = \left(\frac{2n}{n^2+1}\right)c \tag{35}$$

which never exceeds c. It is thus possible to define an energy transport velocity that is never superluminal, although, as noted by Diener [52], this velocity is more 'interpretive' than measurable.

Peatross *et al* [54] have taken a different approach based on a measurable quantity, the Poynting vector. They define the pulse arrival time expectation integral

$$\langle t\rangle_{r} \equiv \frac{\hat{u}\cdot\int_{-\infty}^{\infty} t\boldsymbol{S}(\boldsymbol{r},t)\,\mathrm{d}t}{\hat{u}\cdot\int_{-\infty}^{\infty} \boldsymbol{S}(\boldsymbol{r},t)\,\mathrm{d}t}, \tag{36}$$

where $\hat{u}$ is the unit vector in the direction in which the energy flux is detected. In the frequency domain the expected arrival time has the form

$$\langle t\rangle_{r} = T[\boldsymbol{E}(\boldsymbol{r},\omega)] \equiv -\mathrm{i}\frac{\hat{u}\cdot\int_{-\infty}^{\infty} \partial\boldsymbol{E}(\boldsymbol{r},\omega)/\partial\omega\times\boldsymbol{H}^*(\boldsymbol{r},\omega)\,\mathrm{d}\omega}{\hat{u}\cdot\int_{-\infty}^{\infty} \boldsymbol{S}(\boldsymbol{r},\omega)\,\mathrm{d}\omega}. \tag{37}$$

Based on this expression, and without any essential approximations, Peatross *et al* [54] show that the time delay $\Delta t \equiv \langle t\rangle_{r+\Delta r} - \langle t\rangle_{r}$ associated with the propagation of a pulse from $\boldsymbol{r}$ to $\boldsymbol{r}+\Delta\boldsymbol{r}$ can be expressed as the sum of two distinctly interpretable terms[5]:

$$\Delta t = G_{r+\Delta r} + R_{r}. \tag{38}$$

G_{r}, which Peatross *et al* call the *net group delay*, is given by

$$G_{r+\Delta r} = \frac{\hat{u}\cdot\int_{-\infty}^{\infty} \boldsymbol{S}(\boldsymbol{r},\omega)[\partial(\Re\boldsymbol{k})/\partial\omega]\cdot\Delta\boldsymbol{r}\,\mathrm{d}\omega}{\hat{u}\cdot\int_{-\infty}^{\infty} \boldsymbol{S}(\boldsymbol{r},\omega)\,\mathrm{d}\omega}, \tag{39}$$

i.e. the propagation length divided by the average over all frequencies of the inverse of the group velocity. The *reshaping delay* is given by

$$R_{r} = T[\mathrm{e}^{-\mathrm{Im}\,(\boldsymbol{k})\cdot\Delta\boldsymbol{r}}\boldsymbol{E}(\boldsymbol{r},\omega)] - T[\boldsymbol{E}\boldsymbol{r},\omega)], \tag{40}$$

i.e. the difference between the expected pulse arrival times at the initial point $\boldsymbol{r}$ with and without the change in spectral amplitude due to propagation in the medium. In particular, this reshaping delay vanishes if the pulse spectrum does not change upon propagation.

Peatross *et al* [54] present results of numerical computations for the propagation of Gaussian pulses in a medium described by the Lorentz model with absorption. For pulse bandwidths small compared with the absorption linewidth they obtain results consistent with those of Garrett and McCumber [27]; i.e., the delay time is dominated by the net group delay and can correspond to superluminal and negative propagation velocities. For broadband pulses the reshaping delay becomes important, and in the extreme broadband limit of a delta-function pulse the net group delay is dominant and $\Delta t \to \Delta r/c$ (if $n(\omega) \to 1$ as $\omega \to \infty$, as discussed in section 3). The latter result is consistent with the fact that a sharp wavefront propagates with velocity c.

7. Quantum limitations

Chiao *et al* [55] have suggested that superluminal group velocities could be observed in optical amplifiers whose relaxation times are long compared with the pulse duration, and that this can occur even at the single-photon level. They refer to a single-photon wavepacket propagating

[5] The subscripts $\boldsymbol{r}+\Delta\boldsymbol{r}$ and $\boldsymbol{r}$ can be interchanged in equation (38) without affecting Δt [54].

with superluminal group velocity as an 'optical tachyon'. To avoid spontaneous emission noise they required that the radiative lifetime be large compared with the propagation time through the amplifier, which in turn should be large compared with the pulse duration. In order to avoid superfluorescence (SF), the time delay before the peak of the SF pulse should be larger than all these other times.

Aharonov, Reznik and Stern (ARS) [56] have argued from general considerations and a model involving coupled inverted pendula that 'unstable systems such as media with inverted atoms' have unstable modes whose quantum fluctuations will overcome a superluminal pulse at the single-photon level. This conclusion is based on the reconstruction of the superluminal pulse from a small tail as discussed earlier (figure 5). If the incident field is very weak, the small tail whose analytic continuation determines the propagated pulse can have a very small probability of containing even a single photon, and consequently the superluminal pulse will have a very small signal-to-noise ratio (SNR). One of the conditions imposed by ARS for the observation of superluminality is that $(v_g/c - 1)L/c \gg \tau_p$, where L is the length of the amplifier and τ_p is the pulse duration: the separation in time of pulses propagating with velocities c and v_g must be large compared with the pulse duration in order to strongly distinguish experimentally between luminal and superluminal propagation.

The quantum noise in the ARS model corresponds to SF in a medium of inverted two-level atoms [57]. Even at times short compared with the delay time for the buildup of the peak of the SF pulse, the initial onset of SF is sufficient to make the SNR very small at the one- or few-photon level when the ARS condition of strong distinguishability is imposed. However, the results are not inconsistent with the predictions of Chiao *et al*: even at the one-photon level it is possible to have SNR > 1 if the temporal separation $(v_g/c - 1)L/c$ is smaller than the pulse duration, i.e. if we do not impose the ARS condition but instead allow for a weaker degree of experimental 'distinguishability'. $(v_g/c - 1)L/c < \tau_p$ does not, of course, preclude the observation of superluminal propagation, and in fact this was the case in the tunnelling experiments of Chiao *et al*. As far as the author is aware, $(v_g/c - 1)L/c < \tau_p$ in *all* experiments in which superluminal group velocities have been observed thus far.

Quantum noise associated with spontaneous emission also acts to effectively retard the measured pulse by producing a background level of irradiation that must be exceeded before it can be asserted that the pulse has arrived. We can define a 'signal' arrival time operationally as the time at which the integrated photocurrent exceeds the background level by some predetermined factor. More precisely, the observable corresponding to this definition of the arrival time is [58]

$$\hat{S}(L, t) = \eta \int_{t_0 - T/2}^{t} \mathrm{d}t' \, \hat{E}^{(-)}(L, t)\hat{E}^{(+)}(L, t') \tag{41}$$

at the exit point ($z = L$) of the medium, and we define the arrival time as the time at which the optical SNR [59, 60]

$$\mathrm{SNR}(L, t) = \frac{(\langle \hat{S}_1(L, t)\rangle - \langle \hat{S}_0(L, t)\rangle)^2}{\langle \Delta^2 \hat{S}(L, t)\rangle} \tag{42}$$

reaches a prescribed threshold level. Here $\hat{S}_1$ and $\hat{S}_0$ are defined by equation (41) with and without the input pulse, respectively. $t_0 = T_c + L/c$, where T_c is the time corresponding to the peak of the pulse, T is a time window of typically a few times the pulse width and η is an efficiency factor that is unimportant for the present discussion. This definition of a 'signal' arrival time is obviously somewhat arbitrary, but not unreasonable. When computations are performed using parameters appropriate to the gain medium in the experiments of Wang *et al* [38], it is concluded that the corresponding velocity of the operationally defined 'signal' is less than c [58].

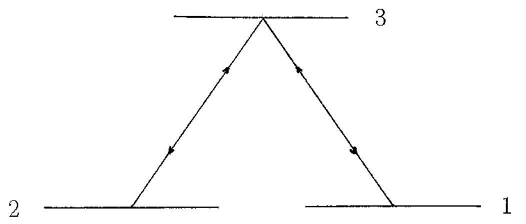

Figure 7. Three-state atom in which the two allowed transitions $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ have the same transition frequency and electric dipole moment.

Superluminal propagation in an absorber would not be affected by quantum noise associated with spontaneous emission. Even in this case, however, the 'arrival time' cannot be inferred before the detection of the first photon. The photon wavepacket (probability amplitude) in any case propagates causally, even though a photon detector can be expected to 'click' earlier than it could if the wavepacket were propagating in vacuum.

8. Highly subluminal group velocities

The observation of pulse propagation velocities very much less than c is not new: experiments in self-induced transparency, for instance, revealed group velocities $\approx$3 orders of magnitude less than c [61]. But the recent observations of *extremely* small group velocities are of a different character in that they are based on quantum interference effects that cause greatly reduced absorption and very rapid variation with frequency of the refractive index.

The interference effect here can be understood using a three-state model with equal transition dipole moments (μ) and equal applied electric field amplitudes ($\mathcal{E}$) at two equal transition frequencies (figure 7). The effect of the applied field on the atom in this model is described by the interaction Hamiltonian

$$H_{int} = -\mu\mathcal{E}[|3\rangle\langle 1| + |3\rangle\langle 2|] + \text{h.c.} \quad (43)$$

Define two superposition states $|C\rangle = |1\rangle + |2\rangle$ and $|NC\rangle = |1\rangle - |2\rangle$. Obviously $\langle 3|H_{int}|C\rangle = -2\mu\mathcal{E}$ and $\langle 3|H_{int}|NC\rangle = 0$.

The 'noncoupled' state $|NC\rangle$ is a *dark state*: an atom in such a superposition state does not interact with the applied field. When the applied field is turned on our three-state atom can be pumped into state 3, from which it can go by spontaneous emission into either the coupled state $|C\rangle$ or the noncoupled state $|NC\rangle$. Once in the latter state it is trapped. Eventually the atom will find itself in the (nonabsorbing) dark state by this process of 'coherent population trapping' [62].

Coherent population trapping in the presence of a coupling field can lead to electromagnetically induced transparency (EIT) [63, 64] for a probe field—that is, the probe can propagate without absorption while the atoms remain unexcited (figure 8). Associated with EIT is a rapidly varying refractive index, and consequently a very small group velocity, and furthermore the refractive index is unity and the group velocity dispersion is zero at line centre [65].

Hau *et al* [66] first observed such extremely small group velocities: $v_g = 17$ m s^{-1} in a Bose–Einstein condensate of sodium atoms. The vanishing of the group velocity dispersion allows a pulse to propagate with essentially no distortion. Among the interesting observations was the spatial compression of the EIT pulse by the factor c/v_g, without a change in the peak amplitude of the electric field. A pulse that would be about 750 m long in free space is observed

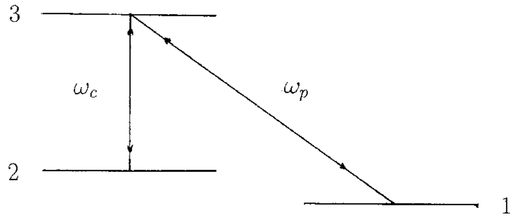

Figure 8. As a consequence of coherent population trapping in the presence of the coupling field (ω_c), a probe field at frequency ω_p can propagate without absorption.

to be compressed to about 42 μm along the direction of propagation in the ultracold gas. This results from energy being transferred from the front part of the pulse to the atoms and the coupling field, and then being returned to the pulse at its back part.

Ultracold temperatures are not required for ultraslow group velocities. Kash *et al* [67] observed group velocities $\sim$ 90 m s^{-1} in ^{87}Rb gas at 360 K, and pointed out that Doppler averaging in EIT and related phenomena is mitigated by a two-photon Doppler-free effect that can occur for co-propagating coupling and probe fields. Budker *et al* [68] have reported an 8 m s^{-1} group velocity in room-temperature ^{85}Rb vapor. The ultraslow group velocity in this case arises from an extremely narrow nonlinear magneto-optic resonance.

8.1. Dark states and dark-state polaritons

Let $\Omega(t)$ be the Rabi frequency associated with the coupling field at the frequency ω_c in figure 8, and denote the operator for the slowly varying envelope of the field at the probe frequency ω_p by $\hat{E}(z,t)$. From the Heisenberg equations of motion it follows that

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) = \mathrm{i}gN\hat{\sigma}_{13}(z,t). \tag{44}$$

Here $\hat{\sigma}_{13}$ is the lowering operator for the $1 \leftrightarrow 3$ transition, N is the number density of atoms and g is the atom–probe coupling constant proportional to the transition dipole matrix element μ_{13}. $\Omega(t)$ is a c-number; i.e., the coupling field is treated as a prescribed, classical field.

The right-hand side of equation (44) is determined by the Heisenberg equations of motion for the operators $\hat{\sigma}_{ij}$. If the probe field is sufficiently weak, so that $\hat{\sigma}_{11} \approx 1$, then

$$\hat{\sigma}_{13} \approx -\frac{\mathrm{i}}{\Omega(t)}\frac{\partial}{\partial t}\hat{\sigma}_{12}. \tag{45}$$

$\hat{\sigma}_{12}(z,t)$ can in turn be approximated by $-g\hat{E}(z,t)/\Omega(t)$ in the adiabatic approximation in which Ω varies slowly. Then

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\hat{E}(z,t) \approx -\frac{g^2N}{\Omega(t)}\frac{\partial}{\partial t}\left[\frac{\hat{E}(z,t)}{\Omega(t)}\right]. \tag{46}$$

In the limiting case in which Ω is constant, this implies the group velocity $v_g = c/(1 + g^2N/\Omega^2)$ [69]. This approximation describes rather well the group velocity observed in the experiment of Hau *et al* [66].

The operator [70]

$$\hat{\Psi}(z,t) = \cos\theta(t)\hat{E}(z,t) - \sin\theta(t)\sqrt{N}\hat{\sigma}_{12}(z,t), \tag{47}$$

where

$$\cos\theta(t) = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + g^2N}}, \qquad \sin\theta(t) = \frac{g\sqrt{N}}{\sqrt{\Omega^2(t) + g^2N}}, \tag{48}$$

satisfies the equation of motion

$$\left[\frac{\partial}{\partial t} + c\cos^2\theta(t)\frac{\partial}{\partial z}\right]\hat{\Psi}(z,t) = 0. \tag{49}$$

This equation has a shape-preserving solution with time-dependent group velocity $v_g(t) = c\cos^2\theta(t)$:

$$\hat{\Psi}(z,t) = \hat{\Psi}\left(z - c\int_0^t \mathrm{d}t'\,\cos^2\theta(t'), t = 0\right). \tag{50}$$

This shape-preserving solution, based on the adiabatic approximation, is closely related to the 'adiabatons' discussed by Grobe *et al* [69]. Note that $\hat{\Psi}$ is neither a pure electric field operator nor a pure atomic operator, but a combination of the two. It is easily shown to have bosonic commutation relations, and Fleischhauer and Lukin [70] refer to its bosonic quasiparticles (number states) as *dark-state polaritons*. These polaritons involve the operator $\hat{\sigma}_{12}$ associated with a *nonallowed* (Raman) transition, thus distinguishing them from the more familiar polaritons defined in terms of allowed transitions. They are called 'dark-state' polaritons because they do not contain the excited atomic state 3.

8.2. Halting and storage of light

The group velocity of the dark-state polaritons depends on the mixing angle θ between their atomic and photonic components. The fact that this mixing angle depends on $\Omega(t)$ means that the group velocity can be controlled by the coupling field. Thus, if θ is changed from 0 to $\pi/2$, the group velocity changes from c to 0. In this case the dark-state polariton goes from being purely photonic to being purely atomic; the light pulse is effectively halted and coherently stored in the medium. Changing θ back to 0 should result in a 're-acceleration' of the light pulse as the polariton changes from purely atomic to purely photonic [70]. Equation (47) shows that this exchange between atomic and photonic components of the dark-state polariton occurs via the atomic coherence of the Raman transition $1 \leftrightarrow 2$. The storage time of the light when the coupling field is adiabatically turned off is therefore limited not by excited-state relaxation but by the $1 \leftrightarrow 2$ coherence lifetime, which is typically very large.

This remarkable halting, storage and re-acceleration of light pulses was reported by Liu *et al* [71], who used sodium vapour at a temperature $\sim 1\ \mu$K, a density $\sim 10^{13}$ cm^{-3} and a propagation length of 339 μm. Phillips *et al* [72] reported similar observations using rubidium at 70–90 °C, 10^{11}–10^{12} atoms cm^{-3} and a 4 cm path length. Both groups observed light storage for a period ~ 1 ms, approximately the coherence lifetime of their Raman transitions.

Research in the area of 'ultraslow' light is very active, and the reader must consult the current literature for an up-to-date account. In one recent development, Agarwal *et al* [73] have shown how the application of an additional field at the $1 \leftrightarrow 2$ transition in figure 8 can be used to change the group velocity from subluminal to superluminal.

9. Summary

It should by now be evident that light pulses can be made to propagate with essentially any group velocity, including $v_g < 0$, $v_g = 0$ and ∞, and that the concept of group velocity does not necessarily lose any meaning or utility when it is dramatically different from c.

Observations of group velocities greater than the speed of light in vacuum are not inconsistent with Einstein causality because they do not involve any superluminal transmission of information. In particular, the peaks of the input and output pulses are not causally connected (figure 5). This is understandable at the purely classical level, and indeed some of the principal results are already contained in the original work of Sommerfeld and Brillouin. But the recent observations of 'superluminal' light pulses, which are realized by choosing appropriately the pulse carrier frequency and bandwidth for a given propagation medium, or by controlling the medium itself (e.g. to produce a gain doublet), are noteworthy for several reasons. First, they should dispel once and for all the notion that the concept of group velocity loses significance when the group velocity exceeds c or has other 'abnormal' values. Second, they have sharpened our understanding of tunnelling and pulse propagation effects, and have revealed effects that many people find surprising. For instance, it has been shown that single-photon wavepackets can propagate such that single photons can be counted, on average, earlier than they could if their wavepackets were to propagate in vacuum at the velocity c. Finally, it is not inconceivable that effects such as these might have applications. We note that Mitchell and Chiao [74] have demonstrated an electronic bandpass amplifier with the property that the group delay is negative for most frequencies; the output waveform can precede the input waveform by several milliseconds[6].

The fact that waveforms can appear to propagate faster than c is not surprising (section 5.3), but has interesting consequences such as the transmission of Mozart's 40th symphony at 4.7 times the speed of light [40]. There is no violation of Einstein causality implied by these experiments or in the experiments on 'superluminal' Bessel beams. The faster-than-c results of the latter experiments are reminiscent of the 'scissors paradox' (section 5.5).

The 'spooky action at a distance' suggested by EPR correlations does not allow any viable schemes for superluminal communication. Such schemes fail for reasons that are themselves quantum mechanical, such as the impossibility of cloning an arbitrary quantum state or the closely related impossibility of building noiseless amplifiers. It is nevertheless fascinating that quantum mechanics seems to go 'right at the border line of contradicting special relativity, but does not cross it' [10].

Experiments based on quantum interference and EIT demonstrate that the group velocity of light pulses can be controlled to such a degree that a pulse can be stopped, stored and 're-accelerated'. The storage time is limited by the coherence time associated with a nonallowed transition rather than by a typical excited-state lifetime, and can therefore be quite long. The preservation of coherence over long times as a pulse is stopped and regenerated suggests that 'ultraslow' light might find applications in areas such as interferometry or 'quantum computation'.

Acknowledgments

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[6] The input and output voltages are related as in section 2.1, with the transfer function $g(\omega)$ given in terms of the inductance, resistance and capacitance of an LRC circuit. The output voltage is related to the input voltage by $V_{\rm out}(t) \approx g(\omega_0)V_{\rm in}(t-t_g)$, where ω_0 is the carrier frequency of the input pulse and the group delay $t_g = (\partial\phi/\partial\omega)_{\omega_0}$. ϕ is the phase of the transfer function and varies with ω in a manner similar to the refractive index near a resonance. Using pulses that turn on or off sharply, Mitchell and Chiao demonstrate that negative group delays t_g are not in conflict with the causality demanded by the Kramers–Kronig relations (or, in the context of electronic circuits, what are called Bode's relations).

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