The Strong Consensus Opinion Dynamics On Adaptive Networks

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The Strong Consensus Opinion Dynamics On Adaptive Networks

Outline

- Sznajd model and its application
- Sznajd model on adaptive network
The dynamics on the network

- Cascading failure: BTW Model, BS Model, OFC Model
- Spreading process: SIS Model, FHN Model, DK Model
- Opinion formation: NG Model, Voter Model, Sznajd Model

S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, D.-U. Hwang
Physics Reports 424, 175 (2006)
Solomon E·Asch [1907.09.14－1996.02.20]:
• Experiment of conformity(1956);
  http://en.wikipedia.org/wiki/Solomon_Asch

Stanley Milgram [1933.08.15－1984.12.20]:
• Behavioral study of obedience(1961);
• Six degrees of separation(1967);
  http://www.stanleymilgram.com/references.php
The Strong Consensus Opinion Dynamics On Adaptive Networks

- Voter model
- Majority Rule model
- Sznajd model
- Deffuant model
- Hegselmann-Krause model

\[ O_{i,t+1} = O_{i,t} + \mu(O_{j,t} - O_{i,t}) \]
\[ O_{j,t+1} = O_{j,t} + \mu(O_{i,t} - O_{j,t}) \]

Hegselmann and Krause, JASSS, 5,3(2002)


The Sznajd model based on social validation

YES = +1

NO = -1

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How does it work?

-----Sznajd model and its application
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A sample simulation

Number of simulation steps:
0 10 100 1000 10000

Voters:
0 20 40 60 80 100

-----Sznajd model and its application
Evolution of the system: social opinion (Yes-No)

\[ m = \frac{1}{N} \sum_{i=1}^{N} S_i \]

What has happened?
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We follow one person ...

---Sznajd model and its application
Characteristic time of opinion change does not exist!

\[ P(\tau) \sim \tau^{-1.5} \]
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The Application of Sznajd model

- On Complex Networks
- Variate Sznajd Rule
- Updating Ways
- Adding Noise
- Adding External Field
- Changing the number of Variable’s number
- Others…..

Claudio Castellano, Santo Fortunato, Vittorio Loreto
REVIEWS OF MODERN PHYSICS, 81, 591 (2009)
The Strong Consensus Opinion Dynamics On Adaptive Networks

The opinion dynamics on adaptive network

- Sznajd model on adaptive network

- Voter model on adaptive network

- Majority Rule model on adaptive network

- Deffuant model on adaptive network

Voter model on adaptive network

\[ \frac{d\rho}{dt} = \sum_k P_k \frac{d\rho}{dt} \bigg|_k = \sum_k P_k \frac{2}{\mu k} [(1 - p)k\langle n \rangle - 2\langle n^2 \rangle] - p\langle n \rangle \]

\[ p_c = \frac{\mu - 2}{\mu - 1} \]

In this model, $N$ agents ($i = 1, \ldots, N$) are endowed with a continuous opinion $O_i$ which can vary between 0 and 1 and is initially random. At each time step $t$, a node $i$ and one of its neighbors $j$ are chosen at random:

- With probability $p$, an attempt to break the connection between $i$ and $j$ is made: if $|o_i - o_j| > d$, a new node $k$ is chosen at random and the link $(i, j)$ is rewired to $(i, k)$.
- With probability $1 - p$ on the other hand, if $|o_i - o_j| < d$ the opinions evolve according to:

$$o(i, t + 1) = o(i, t) + \mu(o(j, t) - o(i, t)) \quad o(j, t + 1) = o(j, t) - \mu(o(j, t) - o(i, t))$$

Majority Rule model on adaptive network

In this network, each node $j$ carries a spin $O_j$ which can take two different values $O_j = \pm 1$. At each time step,
- the links are updated as follows: two nodes carrying equal (unequal) spins are connected with probability $p(1-p)$.
- the spins are updated random sequentially based on a simple majority rule: if they are connected to more positive than negative spins, their state will be positive in the next time step, and negative otherwise; in the case of a tie, the spin remains unchanged.

\[ \partial_t |v(t)\rangle = \mathbb{L} |v(t)\rangle, \]

\[ \partial_t P(M, t) = b_{M-1} P(M - 1, t) + d_{M+1} P(M + 1, t) - [b_M + d_M] P(M, t) \]

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Sznajd model on adaptive network

The network of $N$ ($i = 1, \ldots, N$) agents with $M$ edges are initially connected with each other at random, and opinions $O$ are assigned to nodes which can vary between 0 and 1 randomly. On each step we pick a node $i$, and one of its neighbors $j$ is chosen at random:

- With probability $p$, an attempt to break the connection between $i$ and $j$ is made: if $|O_i - O_j| > d$, a new node $k$ is chosen at random and the link $(i, j)$ is rewired to $(i, k)$.
- With probability $1-p$ on the other hand, the node pair influences to all of its neighbors: if $|O_i - O_j| < d$, the opinions of the neighbors of the pair $i$ and $j$ evolve according to:

$$s_{\Gamma(i),t+1} = (s_{i,t} + s_{j,t})/2; \quad s_{\Gamma(j),t+1} = (s_{i,t} + s_{j,t})/2$$

where $\Gamma(i)$ is the set of neighbors of node $i$.

---

Evolution of the opinions of 20% of the population, denoted by lines, for a system of $10^3$ agents with tolerance $d = 0.15$ and average degree $k = 5$, on a static network $p=0$ (left) and adaptive network $p = 0.7$ (right) for a single run.
Consensus formation on static networks

Probability distribution of the final surviving opinion for continuous Sznajd model on network with
N = 1000 and average degree k = 5 for the 10 000 different initial samples.
Number of clusters (left) and Size of the largest clusters (right) in the final state as a function of the tolerance of the agents for different rewiring rates on adaptive network with average degree $k = 5$ and $N = 1000$ for averaged 100 samples.
Consensus formation on adaptive networks

Size of the largest clusters (left) and number of clusters (right) in the final state as a function of the tolerance value on the adaptive network for different rewiring rates on adaptive network with average degree $k = 5$ and $N = 500$ (black), 1000 (red), 2000 (green), 5000 (blue) for averaged 100 samples.
The relaxation time as a function of the tolerance of the agents for different rewiring rates on adaptive network with average degree \( k = 5 \) and \( N = 1000 \) for averaged 100 samples.
Conclusion

- In the static network, the dynamic will evolve into the state of final one cluster with all agents sharing the same opinion.

- In adaptive network, the opinion cluster coincides with the topology cluster and the final state is a large cluster with a few small clusters.
Time to say „goodbye”
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- Solomon E. Asch: Experiment of conformity (1956)
- Stanley Millgram: Behavioral study of obedience (1961)
- The Application of Sznajd model
Solomon E. Asch, born in Warsaw in 1907, he came to the United States in 1920 and received a Ph.D. from Columbia University in 1932.

Experiment of conformity (without Authority), 1956
Asch (1951-1955) – the experiment on visual perception

The experimenter asks to choose which of the three lines on the left matches the length of the one on the right.
Results of the Asch experiment

- 76% conformed to the majority at least once.
- Conformity became more frequent as group size increased.
- “The tendency to conformity in our society is so strong that reasonably intelligent (…) people are willing to call white black” /Asch/
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Results of the Asch experiment

![Graph showing the percentage of mistakes against the number of people against the majority opinion. The graph shows a sharp increase in mistakes as the number of people against the majority opinion increases from 1 to 3, then levels off as the number of people increases further.](image)

Mistakes against Number of people against
"All those in favor say 'Aye.'"
"Aye."
"Aye."
"Aye."
Stanley Millgram: Obedience to Authority (Yale, 1961-62)
<table>
<thead>
<tr>
<th>volts</th>
<th>learner</th>
<th>experimenter</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>Ou!</td>
<td>Please continue</td>
</tr>
<tr>
<td>105</td>
<td>Ou! (lauder)</td>
<td>It is essential that we continue....</td>
</tr>
<tr>
<td>120</td>
<td>Ou! It hurts!</td>
<td>Continue, it’s necessary</td>
</tr>
<tr>
<td>135</td>
<td>Ou! It is really painful</td>
<td>The experiment requires that you go on until he has learned all the word pairs correctly</td>
</tr>
<tr>
<td>150</td>
<td>Let me leave! My heart ...</td>
<td>You have no other choice</td>
</tr>
</tbody>
</table>
Results of the Millgram experiment
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The Application of Sznajd model

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• On Fully Connected Network

• On Deterministic Pseudo-Fractal Network

• On Albert-Barabasi Network
The Strong Consensus Opinion Dynamics On Adaptive Networks

On Fully Connected Network

\[ m = \frac{N_+ - N_-}{N}. \]

\[
\text{Prob}\left\{ m \rightarrow m + \frac{2}{N} \right\} = \frac{(1 - m^2)}{8} \left( 1 + m + \frac{1 + 3m}{N} \right),
\]

\[
\text{Prob}\left\{ m \rightarrow m - \frac{2}{N} \right\} = \frac{(1 - m^2)}{8} \left( 1 - m + \frac{1 - 3m}{N} \right).
\]

\[
\text{Prob}\left\{ m \rightarrow m \right\} = 1 - \frac{(1 - m^2)}{4} \left( 1 + \frac{1}{N} \right)
\]

\[ t = 2N \tau. \]

\[
\frac{\partial}{\partial \tau} P_m(m, \tau) = -\frac{\partial}{\partial m} \left[ (1 - m^2) m P_m(m, \tau) \right]
\]

\[ P(x, \tau) = [(1 - x^2)x]^{-1} f \left( e^{-\tau} \frac{x}{\sqrt{1 - x^2}} \right) \]
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On Deterministic Pseudo-Fractal Network

\[ P_{\text{cum}}(k) \sim k^{1-\gamma}, \]

\[ \gamma = 1 + \ln 3 / \ln 2. \]

\[ \bar{C} = 4/5. \]

\[ \bar{l} \sim \ln N. \]

\textbf{Fig. 1.} The first three generations of the scale-free pseudo-fractal graph. At each iteration step \( t \), every edge generates an additional vertex, which is attached to the two vertices of this edge.

(1) The network grows, i.e., \( 3^t \) new sites are added.

(2) A random opinion \( (\pm 1) \) is set to each new node of the network, with probability \( p \ (1 - p) \) for opinion +1 \((-1)\).

(3) \( N_s \) Sznajd runs are performed. For each run, \( 3^t \) sites chosen randomly are analyzed and updated, i.e., one visits for the Sznajd model a number of sites equal to the number of sites added at that step to the network.

- \textbf{1 site convincing:}
- \textbf{2 sites convincing:}
Monte Carlo simulations of the Sznajd model

Fig. 2. Sznajd model on a 29,576 nodes pseudo-fractal network with $N_s = 1, 10$ and 50 runs per time step for (a) 1 node convincing, (b) 2 nodes convincing, and (c) 3 nodes convincing.
The Strong Consensus Opinion Dynamics On Adaptive Networks

Renormalization Approach

![Graph showing the renormalization approach](image)

**Fig. 2.** Comparison between the function presented in Eq. 6 (solid line) with Monte Carlo simulations on a growing pseudo-fractal (triangles with error-bars) and on a growing BA scale-free network (stars). In both networks, 29576 nodes are considered. We count the number of samples, out of 1000, for which the fixed point all “up” is obtained when different values for the initial concentration $p$ of nodes “up” are simulated for rule $r = 2$. 
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On Albert-Barabasi Network

- **One site convincing**: For each site $i$ chosen, we change the opinion of all its neighbors to the site's opinion.
- **Two sites convincing**: For each site $i$ chosen, we select randomly one of its neighbors. If this selected neighbor has the same opinion as the site $i$, then all their neighbors follow the pair's opinion. Otherwise, nothing is done.
- **Three sites convincing**: For each site $i$ chosen, we select 2 of its neighbors at random. If all these three sites have the same opinion, they change the opinion of all their neighbors.

![Graphs showing consensus dynamics](image-url)
In the first stage, we started with all the sites with value zero, meaning that there are no committed voters. Then, we visit all the sites exactly once, in random order. For each visit, we try to convince the voter to adopt a candidate, chosen at random. A random number $r$ is generated and compared with $P_c$. If $r \leq P_c$ the candidate is accepted by that voter. If the candidate convinces the voter, this voter tries to convince the neighbouring sites. Once again, we throw the dice and compare a new random number with $P_c$. If successful, $r \leq P_c$, the voter will try to convince the neighbourhood as follows: We check all the six neighbouring sites: for each that has the same value of the candidate chosen before, all the ten neighbouring sites of this bond of two sites will assume the same value (as in the usual Sznajd prescription). If nobody has chosen the same candidate, only the originally selected voter is committed to this candidate.

In the second stage, a usual Sznajd process is performed without using the complication from the probability $P_c$. (We thus assume all voters to be equal and restrict the probability $P_c$ to describe the convincing power of the candidates only.) We go to random sites on the lattice. A neighbouring site is chosen at random and we check if the two sites have the same value (they prefer the same candidate). In that case, all the ten neighbours change to vote in that candidate.

**Fig. 2.** Distribution $N(v)$ for half a million nodes on the Barabasi network, where each previously added node bonds to five previously added nodes. Election of 1000 candidates (+). The number of votes in real elections (×: state of Minas Gerais in Brazil 1998) is multiplied by ten for better comparison.
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Variate Sznajd Rule

a) 如果 $s_i \cdot s_{i+1} = +1$, 则 $s_{i-1}$ 和 $s_{i+2}$ 都与 $s_i$、$s_{i+1}$ 的态度保持一致。

b) 如果 $s_i \cdot s_{i+1} = -1$, 则 $s_{i-1}$ 和 $s_{i+2}$ 保持原有态度不变。

第一类:

a) 如果 $s_{i-1}s_{i+2} = 1$, 则 $s_i$ 和 $s_{i+1}$ 都与 $s_{i-1}$, $s_{i+2}$ 的态度保持一致；

b) 如果 $s_{i-1}s_{i+2} = -1$, 则 $s_i$ 和 $s_{i+1}$ 保持态度不变。

$$E(m) = \frac{1}{2} \left(1 + \frac{H(m)}{H(1)}\right)$$

$$2mNE' (m) - (m^2 - 3) E'' (m) = 0$$

第二类:

a) 如果 $s_i s_{i+1} = 1$, 则 $s_{i-1}$, $s_{i+2}$ 都与 $s_i$ 和 $s_{i+1}$ 的态度保持一致；

b) 如果 $s_i s_{i+1} = -1$, 则 $s_i$ 和 $s_{i+1}$ 分别采取 $s_{i-1}$ 和 $s_{i+2}$ 的态度。

$$E(m) = \frac{1}{2} \left(1 + \frac{Erf\left(m\sqrt{\frac{N}{2}}\right)}{Erf\left(\sqrt{\frac{N}{2}}\right)}\right)$$

其中，$Erf(z) = 2/\sqrt{\pi} \int_0^z e^{-t^2} dt$。

第三类:

$$s_{i+1} = s_i$$

$$s_{i-1} = s_i \quad \& \quad s_{i+1} = s_i$$

如果 $s_i s_{i+1} = -1$, 则 $s_{i-1}$ 和 $s_{i+2}$ 分别采取 $s_{i+1}$ 和 $s_i$ 的态度。

$$E'' (m) = 0$$

$$E(m) = \frac{1}{2} (m + 1)$$
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第一类

$$2mNE'(m) - (m^2 - 3)E''(m) = 0$$

$$E(m) = \frac{1}{2} \left( 1 + \frac{H(m)}{H(1)} \right)$$

其中，$$H(m) = \frac{2^\frac{N}{2}}{2^\frac{1}{2}} \Gamma \left( \frac{3}{2} + N \right) / \Gamma (-N) m \int_0^1 t^{-N-1} (1 - t)^{N+\frac{1}{2}} (1 - tm)^{-\frac{1}{2}} dt.$$  

第三类

$$E''(m) = 0$$  
$$E(m) = \frac{1}{2} (m + 1)$$

其中，$$Erf(z) = 2/\sqrt{\pi} \int_0^z e^{-t^2} dt.$$
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Adding External Field

Advertising: with probability $|h|$ buy product $\text{sgn}(h)$. 

For $h=1$
Who wins?

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Probability of "conquering" the market vs. Power of advertisement (h)

- $c_0=0.05$
- $c_0=0.15$
- $c_0=0.25$