

Optimal transport on supply-demand networks

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Previously, transport networks are usually treated as homogeneous networks, that is, every node has the same function, simultaneously providing and requiring resources. However, some real networks, such as power grid and supply chain networks, show a far different scenario in which the nodes are classified into two categories: the supply nodes provide some kinds of services, while the demand nodes require them. In this paper, we propose a general transport model for those supply-demand networks, associated with a criterion to quantify their transport capacities. In a supply-demand network with heterogenous degree distribution, its transport capacity strongly depends on the locations of supply nodes. We therefore design a simulated annealing algorithm to find the optimal configuration of supply nodes, which remarkably enhances the transport capacity, and outperforms the degree target algorithm, the betweenness target algorithm, and the greedy method. This work provides a start point for systematically analyzing and optimizing transport dynamics on supply-demand networks.

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I. INTRODUCTION

Network transport has attracted increasing attention in recent years (see the review articles [1, 2] and the references therein). Indeed, it describes a large number of natural phenomena and technological processes, such as substance flow in a metabolic network, power transmission in an electric network, information propagation in the Internet, and so on. A matter of prime importance is to make the transport processes more effective and efficient, corresponding to maximizing the global capacity and minimizing the average delivery time. Previous works addressed this issue can be roughly classified into two categories: one concerns the optimization/modification of underlying topology [3–5], while the other focuses on the design of highly efficient transport/routing protocols [6–10].

A latent assumption in most previous works is that every node in a transport network plays the role of a host, that is to say, every node has the ability creating a certain kind of substance, energy or information. However, the real world is far from this assumption. For example, in an electric network [11, 12], there are two kinds of nodes, power stations and transformer substations. The power is generated in the former nodes, flowing to the latter ones, and then imported to customers through them. Therefore, power stations behave as a kind of suppliers, while the transformer substations are customers holding demands. In some Internet serving systems, such as music libraries (e.g., audioscrobbler.com, see Ref. [13]), movie-sharing systems (e.g., Netflix.com, see Ref. [14]) and

on-line viewing site (e.g., YouTube.com, see Ref. [15]), all the resources are located in a few servers, while other connected nodes, usually personal computers, only regale themselves with those services. Those examples give rise to a general concept of *supply-demand network*, whose nodes are classified into two categories: the supply nodes provide some kinds of services, while the demand nodes play the role of customers. Analysis of supply-demand networks has found its applications in various real systems, ranging from the power grid [16, 17] to supply chain networks [18, 19].

In this paper, we propose a general model for the transport on a supply-demand network, whose capacity is very sensitive to the locations of supply nodes. By applying a simulated annealing algorithm, we obtained the near optimal locations of supply nodes subject to the maximal network transport capacity. The proposed algorithm performs obviously better than the random selection, degree targeted, betweenness targeted, and greedy methods.

II. MODEL

Considering a network consisted of N nodes, which are classified into two categories: One is called the *supplier* that provides a certain kind of service, the other is called the *customer* who requires this service. Here, the service is an abstract concept and can stand for substance, energy, information, etc. For simplicity, we use the language of the Internet, that is to say, every customer need some information packets (resource), and only the suppliers can generate those information packets. We assume the demands are uniformly distributed, namely each customer needs a unit number of packets (one can simply

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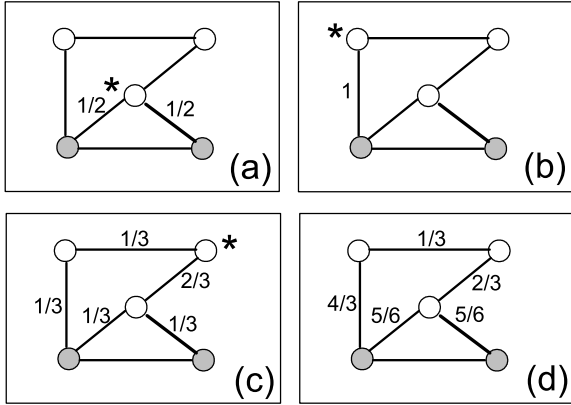


FIG. 1: Illustration of the distribution of edge loads in a supply-demand network. The gray solid and hollow circles denote supply nodes and demand nodes, respectively. In each of panels (a), (b) and (c), the circle marked by a star is the target demand node, and the resulting loads are labeled besides corresponding edges. Integrating (a), (b) and (c), the distribution of edge loads can be obtained, as shown in the panel (d). Here, $L_{\max} = 4/3$.

say one packet). For a given customer, we suppose this packet is always sent by one of the nearest suppliers. However, in general case, there are several nearest suppliers and for each there are several shortest paths. In the real implementation, one of those shortest paths should be randomly picked, and the packet will follow this path from the supplier to the customer. In the numerical calculation, to reduce the fluctuation, if there are in parallel k shortest paths from a customer to the suppliers (generally, those paths aim to more than one nearest suppliers), we assume the packet is divided into k pieces, each goes through one shortest path and contributes $1/k$ to the traffic load (see an illustration in Fig. 1).

If the bandwidth (i.e., traffic capacity) of each edge is identical, the maximal edge load, L_{\max} , is the key factor determining the traffic condition. Actually, the traffic congestion will occur when L_{\max} exceeds the bandwidth. Therefore, given a limited bandwidth, the smaller L_{\max} corresponds to higher transportation capacity. Analogously, in the previous studies [3, 7], the maximal node load is usually used to quantify the system's performance: the smaller the maximal node load, the higher the transport capacity. In this paper, we use edge load instead of node load because in the real systems, such as the Internet and the highway, the congestion usually happens along the edges, not at the nodes [20].

Given a network structure and the number of suppliers, we aim at finding out the optimal configuration of suppliers (i.e., the locations of suppliers) making L_{\max} as small as possible. This is an optimization problem (indeed, an NP hard problem) with L_{\max} being the objective function,

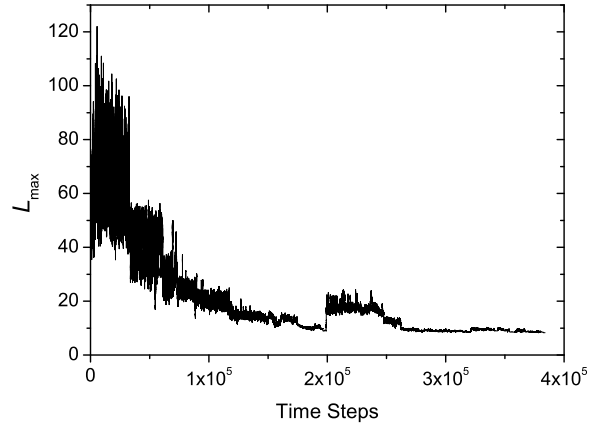


FIG. 2: The objective function, L_{\max} , vs. system time, t , in the optimizing process of the SA algorithm. This figure illustrates a typical result on a BA network of size $N = 1000$, average degree $\langle k \rangle = 6$. The number of suppliers is set as $M = 10$.

and the algorithm presented in this paper (see below) can be directly extended to the case with maximal node load being the objective function. In addition, since many real transportation networks have heterogeneous degree distribution (see the examples shown in Refs. [21, 22]), we use scale-free networks to mimic their topologies.

III. ALGORITHM

In a supply-demand network of N nodes and M suppliers, there are in total $\binom{N}{M}$ different configurations for suppliers' locations. Finding the optimal solution by evaluating all the possible configurations is infeasible when $N \gg M \gg 1$. The optimization of a system with many degrees of freedom with respect to a certain objective function is a frequently encountered task in physics and beyond. One special class of algorithms used for finding the high-quality solutions to those NP-hard optimization problems is the so-called nature inspired algorithms, including *simulated annealing* (SA) [23, 24], *genetic algorithms* (GA) [25, 26], *genetic programming* (GP) [27], *extremal optimization* (EO) [28, 29], and so on. Here we adopt the SA algorithm, whose procedure is as follows.

(i) Randomly choose an initial configuration, denoted by S^0 . Calculate its maximal edge load, L_{\max}^0 , and set the best solution as: $S^{\text{best}} = S^0$ and $L_{\max}^{\text{best}} = L_{\max}^0$. Set the system time as $t = 1$.

(ii) Randomly pick one supplier from the configuration S^{t-1} , and change its location randomly, denote this new configuration as S^t . Calculate its maximal edge load, L_{\max}^t .

(iii) If $L_{\max}^t < L_{\max}^{\text{best}}$, then set $S^{\text{best}} = S^t$ and $L_{\max}^{\text{best}} = L_{\max}^t$. If $L_{\max}^t \leq L_{\max}^{t-1}$, we accept the current configura-

TABLE I: Comparison of the maximal edge load obtained by DTA, BTA, GM and SA. The underlying networks are BA networks with $N = 1000$ and $\langle k \rangle = 6$, and all the data are obtained by averaging over 100 network configurations.

M / Algorithm	DTA	BTA	GM	SA
5	14.73	14.98	13.32	12.37
10	8.25	8.92	7.17	6.31

tion, that is, set $t \leftarrow t + 1$ and repeat (ii). Otherwise, if $L_{\max}^t > L_{\max}^{t-1}$, the current configuration is accepted with probability $e^{-\Delta/T}$, where T is a temperature-like parameter and $\Delta = L_{\max}^t - L_{\max}^{t-1}$. When a configuration is rejected, the algorithm directly goes back to (ii) and keeps the system time t unchanged.

To obtain the high-quality solution, one shall repeat the step (ii) as long as desired. In this paper, we terminate the algorithm if the variance of L_{\max}^t in the latest 10^4 time steps is smaller than a threshold 10^{-6} . Note that, one time step corresponds to one implementation of step (ii), which is different from the system time t . The parameter T is crucial for the algorithmic efficiency. According to the Matropolis's Guidance [30], in the initial stage, the accepting probability of a new configuration should be close to 1. Therefore, we first choose a relatively low temperature T_0 , and numerically calculate the corresponding accepting probability, resulted from a random change of one supplier's location in a completely random configuration. The temperature is doubled until the accepting probability reaches a threshold quantile 0.50. During the searching process, the temperature should slowly decrease [24], here we adopt the simplest method, that is, we set $T \leftarrow \alpha T$ after every Q time steps, where the parameter α is 0.90 and the period is set as $Q = 0.1NM$.

For comparison, we also implement some other algorithms. A brief introduction is as follows. *Random Allocation* (RA) – The locations of suppliers are selected completely randomly. *Degree Target Algorithm* (DTA) – The suppliers are set as the M nodes with highest degrees. *Betweenness Target Algorithm* (BTA) – The suppliers are set as the M nodes with highest betweennesses (see Refs. [31, 32] for the definition and calculation of node betweenness). *Greedy Method* (GM) – First, we consider the case with only one supplier, and find out the optimal location of this supplier that minimizes the corresponding L_{\max} . Then, we add one supplier and find out its optimal location under the condition that the location of the firstly added supplier is fixed. Repeating this operation, that is, at the k th step, we add the k th supplier and find out its optimal location subject to minimal L_{\max} under the condition that the locations of former $k - 1$ suppliers are fixed. This algorithm is terminated when M suppliers are added already.

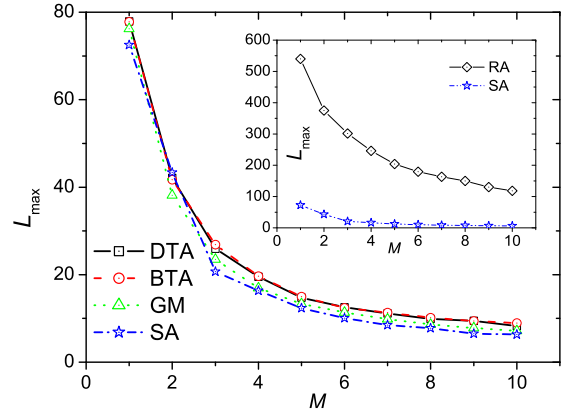


FIG. 3: (Color online) Algorithmic performance for BA networks. The main plot shows a comparison among DTA, BTA, GM and SA, while the inset reports a comparison between RA and SA. The number of suppliers, M , varies from 1 to 10, while the network size $N = 1000$ and the average degree $\langle k \rangle = 6$ are fixed. All the data points are obtained by averaging over 100 network configurations.

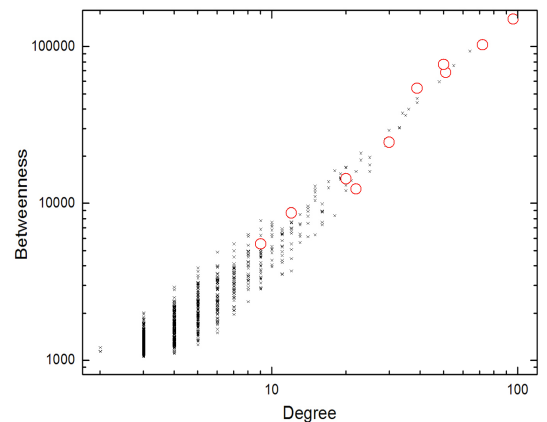


FIG. 4: (Color online) Scatter plot of betweenness *vs.* degree in a BA network with $N = 1000$ and $\langle k \rangle = 6$. Each small black fork represents a node. These 10 red circles denote the selected suppliers by SA. The smallest degree of suppliers is 9, and the second smallest one is 12.

IV. RESULTS

In this paper, all the numerical simulations are implemented based on the Barabási-Albert (BA) model [33], which is one of the minimal models reproducing the heterogeneous structure of real-world networks. Figure 2 reports a typical optimizing process, during which the objective function, L_{\max} , fluctuates strongly in the early stage and approaches to a relatively stable value lately. The proposed SA can reduce the objective function, L_{\max} , by more than 10 times compared with its initial value

corresponding to a random selection of suppliers. We implement SA in larger BA networks ($N = 1000$) for different M from 1 to 10, and take the average over 100 independent network configurations. As shown in the inset of Fig. 3, SA performs much better than RA. We also compare SA with some mentioned algorithms, DTA, BTA and GM, and the results have demonstrated that SA performs best. We report two examples, $M = 5$ and $M = 10$, in Table 1. The improvement is in general about 10%. Note that, although SA performs the best, it spends the longest running time. Actually, the time complexity obeys the inequality $O(SA) > O(GM) > O(BTA) > O(DTA)$. Since GM performs not so bad, it is a strong candidate especially for huge-size networks, and GM might be a considerable tradeoff of time complexity and accuracy of solution.

Note that, although BA model has successfully captured the degree heterogeneity of real networks, it lacks some other important structural properties, such as the community structure [34] and rich-club phenomenon [35]. DTA might perform worse if the network has strongly community structure or presents the rich-club phenomenon. The reason is a good algorithm should prefer to allocate suppliers to different communities rather than putting them together in a community containing many very-large-degree nodes, and if the very-large-degree nodes are closely connected to form a rich club, selecting them as a whole is of low efficiency since the increasing suppliers cannot substantially reduce the average distance from customers to suppliers. As a start point, we here only discuss simulation results on BA networks, and leave the investigations of algorithmic performance on more complicated topologies as an open issue.

The DTA and BTA have almost the same performance and give out very similar selections of suppliers, for in BA networks betweenness and degree are very strongly correlated [36, 37]. To provide insights of the solution by SA, in Fig. 4, we give a scatter plot of betweenness versus degree, and mark by red those selected suppliers. Though SA also prefers large-degree (large-betweenness) nodes, the selected suppliers are remarkably different from those by DTA or BTA, actually, moderate-degree (moderate-betweenness) nodes also have chance to be selected by SA. In most cases, only the top-40% large-degree nodes have the chance to be selected, therefore we can restrict the candidates of suppliers in those 40% nodes. We have checked this restriction in BA networks with $N = 1000$ and $\langle k \rangle = 6$, which gives out equivalently good solution while requires about 10 times shorter CPU time.

V. CONCLUSION AND DISCUSSION

In this paper, we proposed a generic model of transport in supply-demand network, which is consisted of suppliers (supply nodes) and customers (demand nodes). Accordingly, a measure of edge load is given, under the assumption that every customer only requires service from

the nearest supplier. In such a network with heterogeneous degree distribution, its transport capacity is very sensitive to the locations of supply nodes. We therefore design a simulated annealing algorithm to find out the near optimal configuration of supply nodes, which remarkably enhances the transport capacity, and outperforms the degree target algorithm, the betweenness target algorithm, and the greedy method. This work provides a start point for systematically analyzing and optimizing transport dynamics on supply-demand networks. Even though the model and algorithm are simple, we get some non-trivial result, that is, simply picking up those nodes of highest degrees is not the optimal method, actually, some moderate-degree nodes also have chance to be selected as suppliers.

In our model, every customer requires the same amount of resource, which is not in accordance with *the elephants and mice phenomenon* [38] found in the real Internet, where a small fraction of flows contribute to most of the traffic. Corresponding to the current model, a flow stands for the resource transported from a supplier to a customer, and thus each flow has the same size although the one passing longer paths contributes more to the total load. In addition, the proposed algorithm does not fully take into account and make use of the topological features. We have already mentioned in the last section that the mesoscopic structure, such as communities and the rich club, may highly influence the solutions. Those structural information should be extracted prior to the optimizing algorithm, and be embedded in the algorithmic procedure in some way to improve the efficiency and/or the resulting network capacity. All those blemishes listed above can be treated as some open problems worth of a future exploration.

To the end, we emphasize that many real systems can be better described by the current supply-demand network model, instead of the oversimple assumption [7] that every node simultaneously plays the roles of supplier and customer. We have already mentioned some examples, such as power grid [16, 17] and supply chain networks [18, 19], another typical example is the software supporting systems in the Internet, where a system usually has set up several servers in different locations, and users from everywhere can ask for downloading of some softwares. The locations of those servers play the crucial role in determining the efficiency and capacity the software supporting system.

This study also provides some complementary information for relevant phenomena in disparate systems. For example, social scientists have studied how to design who should be integrators in a given social communication networks to better solve problems, and they have found that people having extensive relations (i.e., of very large degrees) may not be the suitable information integrators, instead, the highest efficient structure makes the distance of all nodes from the obvious integrator the shortest [39], which is, to some extent, in accordance with what we observed in this work. In addition, empirical studies show

that the public service facilities are not just located in the place of the most dense population, but somehow more uniformly distributed to make the total travel distance between people and facilities shorter. As a final remark, we noted that a very recent work has considered of the network-based transport with multiple sources and sinks [41], which shows different yet relevant motivation to the current work.

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