Proceedings of 2010 Conference on Dependable Computing (CDC'2010)
November 20-22, 2010, Yichang, China

Wavelet Image Inpainting Based on Dictionary Learning with a Beta Process

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Abstract **- The problem of image inpainting and wavelet image inpainting were presented in this study. Dictionary Learning with a Beta Process (BPDL) was introduced. A new method based on BPDL was proposed for wavelet image inpainting. Unlike conventional methods which mostly based on diffusion theory in physics, this method is based on sparse image representation and considers an image as a combination of different structural patterns to achieve inpainting. The image simulation experiments were designed to test the algorithm. The results demonstrated that the connectivity principle of human perception was well realized with good vision effect. The PSNR of the wavelet coefficients partly damaged images was improved significantly after processed by the new method. It's also available for NIR images. It's concluded that the presented method based on BPDL was an effective method for wavelet image inpainting.**

Keywords **- Image Inpainting, Wavelet, Dictionary Learning, Beta process**

I. INTRODUCTION

Since it was firstly introduced by Bertalmio et al. [1], image inpainting has developed quickly and becomes one of the important contents of Digital Image Processing. The purpose of image inpainting is to restore images to satisfy human vision request by filling the lost parts of the damaged images with residual information. Bertalmio et al. developed the Bertalmio-Sapiro-Caselles-Balleste(BSCB) model which adopted the idea of thermal diffusion in physics, since then many of PDE-based models were proposed such as CDD(Curvature-Driven Diffusion) model[2], TV(total variation) model [3], Euler's elastia model[4], Mumford-Shah model[5], Mumford-Shah-Euler model[6]. Furthermore, to inpaint images damaged in large blocks, texture synthesis was introduced to deal with those damaged areas[7]. The large damaged blocks could be divided into construction and texture, and the former can be inpainted using methods mentioned above, while the latter would be processed by texture synthesis.

In the last few decades, wavelet analysis has developed extensively and influenced both mathematics and applied subjects[8]. Wavelet transform, which is characterized by analyzing signal in time-frequency field multiresolution analysis, has become a useful tool to analyze signals and images. With the maturity of wavelet theory, wavelet has been widely applied to image processing, such as compression [9], denoising[10] and enhancing[11]. During the process of image processing by wavelet, some of wavelet coefficients will be lost due to various reasons, and finally result in damaged images in pixel domain after reconstruction.

Image representation has become a hot spot in image processing. Sparse image representation based on over-complete dictionary is a new method for image representation[12]. Various structural characteristics can be captured through the redundancy of overcomplete dictionary obtained from training and learning process, in this way the effective image representation could be achieved [13]. Recently, this theory has been widely applied in many fields of image processing for its sparsity, characteristic retentivity, and separability, etc. [14, 15].

Due to many advantages of Dictionary Learning with a Beta Process(BPDL) mentioned in [16], the present study discussed the feasibility of using BPDL to wavelet image inpainting. Traditional approaches for wavelet image inpainting were mostly based on PDE, while the BPDL-based method presented in this study was based on sparse image representation. In section II, Dictionary Learning with a Beta Process(BPDL) was briefly presented. In section III, a method based on BPDL was proposed, and the details for implement were given. In addition, the connectivity principle of human perception was proved. In section IV, some typical image simulating experiments were shown to test the method.

II. DICTIONARY LEARNING WITH A BETA PROCESS

Since Beta Process(BP) was defined by Hjort in 1990, it has been introduced into many fields like factor analysis, modelling hazard rate[17]. Meanwhile, it was extended to be two-parameter BP by Parsley et al.[18]. And it was introduced into dictionary learning by Mingyuan Zhou et al. [16]. The number of the dictionary elements and their relative importance can be non-parametrically inferred, and there's no need to assume a priori knowledge of noise variance when applied to denoising and inpainting. In this study, dictionary learning with a beta process is written as BPDL. BPDL was well used in image processing like denoising, Compressive Sensing(CS), etc.

$BP(a,b,H_0)$ is a Beta Process where $a>0$, $b>0$, H_0 is the base measure. The distribution of $H \sim BP(a,b,H_0)$ can be represented as

$$\begin{aligned} H(\psi) &= \sum_{k=1}^{K} \pi_k \delta_{\psi_k}(\psi) \\ \pi_k &\sim Beta(a/K, b(K-1)/K) \\ \psi_k &\sim H_0 \end{aligned} \tag{1}$$

where $\delta_{\psi_k}(\psi)=1$ if $\psi=\psi_k$, otherwise $\delta_{\psi_k}(\psi)=0$. It has valid measure as $K\to\infty$. Therefore, $H(\psi)$ is a vector of K probabilities which associates with an atom ψ_k. As $K\to\infty$, $H(\psi)$ is an infinite-dimensional vector with each element associates atom ψ_k being independent and identically distributed (i.i.d.) from H_0.

N binary vectors can be drawn from $H(\psi)$. $z_i\in\{0,1\}^K$ denotes the i th of these vectors, and the k th of z_i is drawn $z_{ik}\sim Bernoulli(\pi_k)$. These vectors constitute a matrix $Z\in\{0,1\}^{K\times N}$, where the i th column is z_i, the k th row atom ψ_k.

The thought of signal sparse representation is to substitute redundant bases of overcomplete dictionary for orthogonal bases. The chosen dictionary should better contain the structure of signal as much as possible, so the signal can be optimally linearly combined by some of atoms chosen from the dictionary. Actually, it's a process of approximation.

For a signal $x_i\in \mathrm{D}$, let $\Psi=(\psi_1,\psi_2,\cdots,\psi_K)$ a dictionary, where ψ_i is an atom in Ψ, then the sparse representation model is

$$x_i=\bar{x}_i+\varepsilon_i=\Psi z_i+\varepsilon_i \tag{2}$$

where $\bar{x}_i$ is the approximation of x_i, ε the error term, z_i the coefficient matrix. From the sparse representation perspective, we hope to get the sparsest z_i as ε_i is minimal. The optimization function is constructed as

$$\min\|z_i\|_0 \quad s.t. \quad \|x_i-\Psi z_i\|_2\le\varepsilon_i. \tag{3}$$

The above model is highly restrictive, and the coefficients of expand dictionary should be binary. Therefore the weights $w_i\sim \mathrm{N}(0,\gamma_w^{-1}I_K)$ are drawn, where γ_w is the precision or inverse variance. And thus the dictionary weights are $\alpha_i=z_i\circ w_i$, and $x_i=\Psi\alpha_i+\varepsilon_i$, where $\circ$ is the Hadamard multiplication of two vectors.

Assume atoms ψ_k are drawn from a multivariate Gaussian base H_0, and error vectors ε_i are drawn i.i.d. from a zero-mean Gaussian. The hierarchical form of the model is

$$\begin{aligned} &x_i=\Psi z_i+\varepsilon_i && \alpha_i=z_i\circ w_i\\ &\Psi=(\psi_1,\psi_2,\cdots,\psi_K) && \psi_K\sim \mathrm{N}(0,n^{-1}I_n)\\ &w_i\sim \mathrm{N}(0,\gamma_w^{-1}I_K) && \varepsilon_i\sim \mathrm{N}(0,\gamma_w^{-1}I_n)\\ &z_i\sim\prod_{k=1}^{K}Bernoulli(\pi_k) && \pi_k\sim Beta(a/K,b(K-1)/K)\end{aligned} \tag{4}$$

Consecutive elements in the model are in the conjugate exponential family so they can be inferred via a variational Bayesian or Gibbs-sampling analysis.

III. BPDL-BASED WAVELET IMAGE INPAINTING

Wavelet image inpainting is to minimize the energy functional of damaged images in wavelet domain, which was introduced by F. Chan et al. and they proposed the TV model at the same time[19]. Later a fast optimization transfer algorithm was proposed for wavelet image inpainting by H. Chan et al.[20]. A Total Variation Wavelet Inpainting Model with Multilevel Fitting Parameters was proposed by F. Chan et al.[21]. Nonlocal method was introduced into wavelet inpainting by Xiaoqun Zhang et al.[22].

Let an image $z\in R^{N_y\times N_x}$, $z(x)=u_0(x)+n(x)$, where $u_0(x)$ is the original noiseless image, $n(x)$ the Gaussian white noise with $\|n(x)\|_2=\sigma$. Let the standard transform of $z(x)$ $z(\alpha,x)=\sum_{j,k}\alpha_{j,k}\psi_{j,k}(x)$, $j\in Z,k\in Z^2$. The task of wavelet inpainting is to restore the missing coefficients $\{\alpha_{j,k}\}$ in a proper manner to have the image contained as much information as possible.

Due to the capability in controlling geometrical features of images, the impact of PDE techniques in image processing is growing. Traditional wavelet inpainting models are mostly based on variational PDE. They adopt the thought of thermal diffusion in physics. Let us take TV model as an example. For images with free or neglectable noise, we just need to fill in the damaged wavelet coefficients and preserve the other coefficients. The model is

$$\begin{aligned}\min_{\beta_{j,k}:(j,k)\in I}F(u,z)&=\sum\nolimits_{R^2}|\nabla_x u(\beta,x)|dx\\ &=TV(u(\beta,x))\end{aligned}, \tag{5}$$

where $u(\beta,x)$ has the wavelet transform

$$\begin{aligned}u(\beta,x)&=\sum_{j,k}\beta_{j,k}\psi_{j,k}(x),\\ \beta&=(\beta_{j,k}),j\in Z,k\in Z^2\end{aligned} \tag{6}$$

with $\beta_{j,k}=\alpha_{j,k}$, $(j,k)\notin I$, where $I=\{(j,k)|\alpha_{j,k}\}$ is the missing or damaged wavelet coefficient. The Euler-Lagrange equation for the above model is

$$-\int_{R^2}\nabla\cdot\left[\frac{\nabla u}{|\nabla u|}\right]\psi_{j,k}dx=0. \tag{7}$$

Noise is inevitablely introduced into the loss of data packages in many applications like signal transmission or processing, i.e. $\{\beta_{j,k},(j,k)\notin I\}$ can be polluted by noise while $\{\beta_{j,k},(j,k)\in I\}$ can be completely damaged. In this situation, the model is

$$\begin{aligned}\min_{\beta_{j,k}}F(u,z)&=\int_{R^2}|\nabla_x u(\beta,x)|dx\\ &+\sum_{(j,k)}\lambda_{j,k}(\beta_{j,k}-\alpha_{j,k})\end{aligned}, \tag{8}$$

where $\lambda_{(j,k)}$ is zero if $(j,k)\in I$. Its Euler-Lagrange equation is

$$-\int_{R^2}\nabla\cdot\left[\frac{\nabla u}{|\nabla u|}\right]\psi_{j,k}dx+2\lambda_{j,k}(\beta_{j,k}-\alpha_{j,k})=0. \tag{9}$$

In this paper, BPDL is introduced into wavelet image inpainting. Unlike most conventional wavelet inpainting models, it is not based on diffusion theory in physics. The new model decompose an image into different element sets by which the image is combined i.e. it sees an image as a combination of many structures and characteristics. The details to implement the inpainting algorithm based on BPDL is given below.

Let $\hat{z}$ a layer of wavelet coefficients of image z with partly damaged or missing values. $\hat{z}$ is partitioned into $N_B = (N_y - B + 1) \times (N_x - B + 1)$ blocks $\{x_i\}_{i=1,N_B}$, where $x_i \in R^{B^2}$, $B = 8$ typically. Considering any pixel $[p, j]$, where $p, j \in [1, B]$, and let this pixel constitute the left-bottom pixel in a $B \times B$ block. All the $B \times B$ blocks the left-bottom pixel of which lie at

$$\begin{aligned} &\{p + \ell B, j + mB\} \cup \delta(p-1)\{N_y - B + 1, \\ &j + mB\} \cup \delta(j-1)\{p + \ell B, N_x - B + 1\} \end{aligned} \quad (10)$$

are denoted as D_{pj}, where ℓ and m satisfy

$$p + \ell B \leq N_y - B + 1, \; j + mB \leq N_x - B + 1. \quad (11)$$

For $1 \leq p \leq B$, $1 \leq j \leq B$, there are B^2 those sets.

We use the blocks in D_{11} to do the first iteration to train Ψ, and to initialize Ψ and α_i based on a singular value decomposition(SVD) for the first round. After several Gibbs iterations with D_{11}, Ψ and α_i from the previous step are retained for the next round to initialize the Gibbs sampler. Using the above steps, the algorithm is run on the blocks in $D_{11} \cup D_{21}$. Gibbs sampler is now run on the expended data for several iterations, the last sample is retained, and the data set is augmented again. The blocks of $\hat{z}$ are processed according to the above steps for $B^2 = 64$ times. Wavelet image processing is not done until all layers of wavelet coefficients are processed following the above procedure.

The iteration of BPDL had been discussed and proved to be valid in the reference [17], in which the reader could referred for further details. Therefore, here we just provide those relative details for wavelet inpainting. In fact, the validity is also demonstrated via proving the connectivity principle and image simulations in the next section.

To test if the proposed method realizes the connectivity principle or not, Fig.1 (left) is decomposed by Daubechies 7 biorthogonal wavelets with symmetric extensions at the boundaries. 75% wavelet coefficients in area $(100:150, 100:150)$ is randomly lost. The directly reconstructed image using remaining coefficients is in Fig.1 (middle). The processed image in pixel domain is shown as Fig.1(right). For further details of the process, some layers of wavelet coefficients are also shown below. Fig.2(upper left) is the first layer of high frequency wavelet coefficients. Fig.2(upper right) is the partly damaged wavelet coefficients. Fig.2(bottom left) is the trained dictionary. Fig.2(bottom right) is the wavelet coefficients inpainted by the algorithm. The result demonstrated that the connectivity principle of human perception was well realized.

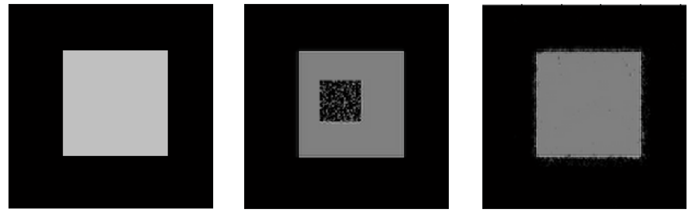

Fig. 1 The proof of the connectivity principle of human perception for the proposed method

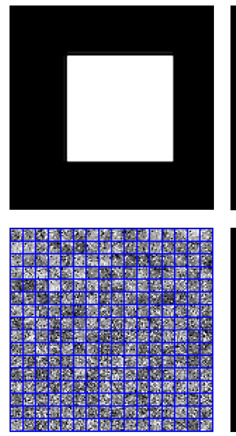

Fig. 2 Some details of inpainting of one layer of wavelet coefficients

IV. EXAMPLES AND VALIDATION

In this study, Standard Peak Signal to Noise Ratio(PSNR) is applied to quantify the improvements of performance of inpainting to test the proposed method. PSNR is defined as

$$PSNR = 10 \log_{10}\left(\frac{255^2}{\|u - u_0\|_2}\right)(dB) \quad (12)$$

where u_0 is the original image, u is the image processed by proposed method. The higher the PSNR, the better the effectiveness of inpainting.

In the experiments, original images are firstly decomposed by Daubechies 7-9 biorthogonal wavelets with symmetric extensions at the boundaries. Then 75% of wavelet coefficients of each image are randomly lost. The damaged images in pixel domain are obtained by reconstructing those damaged coefficients. Fig.3(left) is an noise-free image which contains various simple geometric drawings. Its damaged image in pixel is showed as Fig.3(middle) with $PSNR = 1.28dB$. Fig.3(right) is the result after processed by the inpainting method, whoes $PSNR$ is $15.90dB$. The next group of experiments uses image Lena shown as Fig.4(left). After randomly losing wavelet coefficients, the reconstructed images is Fig.4(middle). Its $PSNR$ increases from $2.15dB$ to $14.18dB$. The performance is as Fig.4(right) shows. Recently NIR imaging has been widely applied in many fields. NIR image processing is an important step during its many applications. When using wavelet to process NIR images, it's unavoidable to lose some coefficients during data transmission. The last group of experiments is to test the method's effectiveness for NIR images. Fig. 5(left) is the original image, middle represents coefficients partly damaged image with reconstructed image's $PSNR = 1.95dB$, right is reconstructed image after inpainted using our proposed algorithm with $PSNR = 15.91dB$.

Simulation result showed that on the basis of realizing the connectivity principle of human perception, the proposed algorithm improved the PSNR of the wavelet coefficients partly damaged images with good vision effect even more significantly compared with conventional methods. It's not only useful for general images but also available for NIR images. Therefore, it's an effective BPDL-based method for wavelet image inpainting.

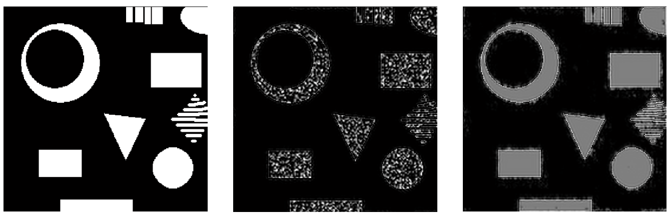

Fig. 3 Lab 1, to inpaint an image contains many simple geometric drawings

Fig. 4 Lab 2, to inpaint image Lena

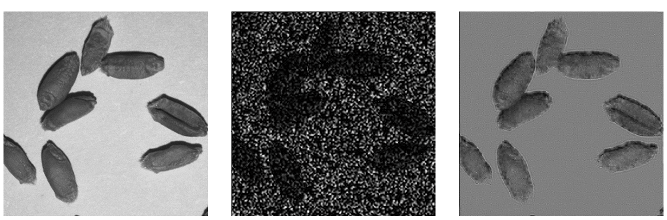

Fig. 5 Lab 3, to inpaint a NIR image of wheat seed

V. DISCUSSION AND CONCLUSION

The problem of image inpainting both in pixel and wavelet domain and their developments are presented in this paper. Dictionary learning with a beta process(BPDL) is also introduced. A new method based on BPDL is proposed to inpaint images whose wavelet coefficients are partly damaged. The simulating experiments demonstrated that on the basis of realizing connectivity principle of human perception the proposed method improved the PSNR of damaged images with good vision effect. Moreover, it's also available for NIR images. It's an effective method to deal with wavelet inpainting problem. However, there are something needed to be done to improve this algorithm, such as image contrast enhancement, texture reservation, speeding up and calculation.

ACKNOWLEDGMENT

This work was supported by the 948 Ministry of Agriculture project(2006-G63(5), 2010-S20).

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