A P.D.E. MODEL OF HIV TRANSMISSION WITH SOCIAL BEHAVIOR

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Abstract. In this study, we given some partial differential equation (P.D.E.) modeling about the social behavior of HIV transmission. By using the Predictor-Corrector method we obtain a numerical solution about the modeling which show that the social behavior can be important to the prediction of the HIV transmission.

1. Introduction

As Human Immune Deficiency Virus (HIV)/Acquired Immune Deficiency Syndrome (AIDS) is projected to remain of critical importance in this century, attempts to forecast and predict its developments are urgently needed. A vast amount of literature[Yus08, Yue07, Q.L02, FK91] describing many different aspects of the disease has already been investigated. Rarely can one find an attempt to model the spread of AIDS incorporating the basic spatial dimensions of human existence. At a global level, there are considerable differences between regions, states, localities, cities, towns and villages in the levels of prevalence, and rate of transmission, of the HIV/AIDS [aid]. There are also differences between the HIV carriers and sufferers which include the relative proportions of heterosexuals/homosexuals, injecting drugs users, male/female infant HIV carriers. We call the factors above the social behaviors. In the following section, we consider a certain given population in nature with spatial moving.

2. The P.D.E. Model with Spatial Moving

As a simple example, a particular point concentration $G_2$ in fixed time will affect the socio-human behavior near and both will influence the HIV spread rates. $G_0$ and $G_{n+1}$ can be the boundary populations. However, from a biological perspective, the diffusion of individuals may be connected with other things, such as escaping high infection risks. Therefore individuals may move along the gradient of infectious individuals to avoid higher infection.

Key words and phrases. P.D.E. system, HIV/AIDS behavior, predictor-corrector method, numerical simulation.
Motivated by these observations, we consider an epidemic model with the modern metapopulation theory. Our model is

\[
\begin{aligned}
\frac{\partial U}{\partial t} &= \Lambda \Delta U + F(x, t, U) \quad x \in \Omega, 0 < t \leq T \\
U(x, 0) &= \Phi(x) \\
U|_\Gamma &= G(t)
\end{aligned}
\]  

\(U = (u_1, u_2, \cdots, u_n)^T\) denotes the concentration of different populations \(G_1, G_2, \cdots, G_n\) at the given environment \(\Omega\) with the boundary \(\Gamma\) and \(u_i = u_i(x, t)(i = 1, \cdots, n, x = (x_1, x_2, \cdots, x_n))\). \(\Lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)^T\) denotes the diffusion coefficients respect to \(U\). \(\Delta U = (\Delta u_1, \Delta u_2, \cdots, \Delta u_n)^T\) where \(\Delta\) is the Laplace operator. \(x = (x_1, x_2, \cdots, x_n)\).

From above equation we know that When \(\Lambda = 0\) and \(F(x, t, U) = F(t, U)\) the model (2.1) becomes an ODE system.

### 3. Numerical Simulation for the P.D.E. Model

We assume the partial differential equation to have the form

\[
u_{xx} = \psi(x, t, u, u_x, u_t)
\]

Predictor-corrector methods have been successfully used by many in the numerical solution of ordinary differential equations. A discussion of some of these is to be found in some books William F. Ames [AME77] and so forth. The general approach begins from known or previously computed results at previous pivotal values, up to and including the point \(x_n\), by ‘predicting’ results at \(x_{n+1}\) with formulae which need no knowledge at \(x_{n+1}\). These predicted results are relatively inaccurate. They are then improved by the use of more accurate ‘corrector’ formulae which require information at \(x_{n+1}\). Generally this amounts to computing results at \(x_{n+1}\) from a nonlinear algebraic equation, to the solution of which the ‘predictor’ gives a first approximation and the ‘corrector’ is used repeatedly, if necessary, to obtain the final result.

Douglas and Jones [BF89] have considered Eqn(3.1) on \(0 < x < 1, 0 < t \leq T\) with \(u(x, 0), u(0, t) and u(1, t)\) as specified boundary conditions. If

\[
\psi = f_1(x, t, u) \frac{\partial u}{\partial t} + f_2(x, t, u) \frac{\partial u}{\partial x} + f_3(x, t, u)
\]

a predictor-corrector modification of the Crank-Nicolson Procedure is possible so that the resulting algebraic problem is linear. The equation (3.2) includes our equation (2.1) when \(x \in \mathbb{R}\).

If \(\psi\) is of the form of (3.2), the following predictor-corrector analog (combined with the boundary data \(u_{i,0}, u_{0,j}\) and \(u_{M,j}\)) leads to linear algebraic equations. The predictor is

\[
\frac{1}{h^2} \delta_x^2 U_{i,j} + \frac{1}{2} = \psi \left[ ih \left( j + \frac{1}{2} \right) k_i U_{i,j}, \frac{1}{2h} \mu \delta_x U_{i,j}, \frac{2}{k} \left( U_{i,j} + \frac{1}{2} - U_{i,j} \right) \right]
\]
for $i = 1, 2, \cdots, M - 1$. This is followed by the corrector
(3.4)
\[
\frac{1}{2} \delta_x^2 [U_{i+1} + U_i] = \psi \left[ i h, (j + \frac{1}{2}) k, U_{i+1} + U_i, \frac{1}{4h} \mu \delta_x (U_{i+1} + U_i), \frac{1}{k} (U_{i+1} - U_i) \right]
\]

Equation (3.3) is a backward difference equation utilizing the intermediate time points $(j + 1/2) \Delta t$. Since Eqn (3.2) only involves $\partial u / \partial t$ linearly, the calculation into the $(j + 1/2)$ time row is a linear algebraic problem. To move up to the $(j+1)$ time row we use Eqn (3.4), and by virtue of the linearity of Eqn (3.2) in $\partial u / \partial t$, this problem is also a linear algebraic problem.

Douglas and Jones [BF89] have demonstrated that the predictor-corrector scheme defined by Eqn (3.3) and (3.4) converges to the solution of Eqn (3.1) when $\psi$ is specified by Eqn (3.2). The truncation error is $O[h^2 + k^2]$. 
4. CONCLUSION

When $x \in \mathbb{R}$ we take the stochastic variables from 0 to 0.3. as the initial concentration of $G_n(n = 0, 1, \cdots, n + 1)$. the numerical solutions of (2.1) shows that there are great differences between the phase graph with spatial motion and the trivial one. the figures 1 and 2 show the different distribution of $U$ with different diffusion coefficients $\Lambda$. In Fig. 1 and Fig. 2, the vertical orient represents the value of $U$.

REFERENCES


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