Theoretical and Experimental investigation of Electromagnetic NDE for defect characterisation

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TABLE OF CONTENTS

TABLE OF CONTENTS .................................................................I
LIST OF FIGURES ........................................................................ IV
LIST OF TABLES .......................................................................... VII
ACKNOWLEDGEMENTS ............................................................. VIII
SUMMARY .................................................................................. IX
ABBREVIATIONS ....................................................................... X

CHAPTER 1 INTRODUCTION ......................................................... 1
  1.1 Research background ............................................................ 1
  1.2 Aims and Objectives ............................................................... 3
    1.2.1 Research aims ............................................................... 3
    1.2.2 Research objectives ....................................................... 3
  1.3 Main achievements .............................................................. 5
  1.4 Thesis layout ....................................................................... 6

CHAPTER 2 LITERATURE SURVEY .............................................. 8
  2.1 ENDE techniques ................................................................. 8
    2.1.1 Eddy Current ............................................................... 9
    2.1.2 Magnetic Flux Leakage .................................................. 12
    2.1.3 Remote Field Eddy Current .......................................... 15
  2.2 Numerical Simulations of ENDE ........................................... 16
  2.3 Analytical Modelling of ENDE .............................................. 19
  2.4 Quantitative ENDE and inverse process ............................... 23
  2.5 Summary and problems identified ....................................... 26

CHAPTER 3 THEORETICAL BACKGROUND OF ENDE ................. 28
  3.1 Maxwell’s equations and deduced governing equations for ENDE ..... 28
  3.2 Numerical approach in solving time-harmonic and time-stepping problems of ENDE 31
  3.3 Analytical approach in solving time-harmonic problems of ENDE ............ 32
  3.4 Experimental investigation of ENDE ..................................... 32
  3.5 Research methodology .......................................................... 35
    3.5.1 Theoretical study ......................................................... 36
      3.5.1.1 FEA for EC, dynamic MFL and RFEC ......................... 36
      3.5.1.2 ETREE modelling for SFEC and PEC ......................... 37
    3.5.2 Experimental study ....................................................... 39
  3.6 Chapter summary .................................................................. 40

CHAPTER 4 FINITE ELEMENT ANALYSIS FOR ENDE ............... 41
  4.1 Case study I: FEA for MFL .................................................... 41
    4.1.1 FE simulations for MFL with irregular-shaped crack .................... 41
      4.1.1.1 Background ........................................................... 42
      4.1.1.2 Simulation setup ..................................................... 43
      4.1.1.3 Simulation results and experiment .............................. 45
      4.1.1.4 Summary of static FEA for MFL ............................... 50
    4.1.2 FE simulations for dynamic MFL inspection .......................... 51
      4.1.2.1 Simulation setup ..................................................... 51

1
6.3.3 LOI range vs. conductivity of the sample .................................................. 118
6.4 Inverse scheme using LOIs derived from introducing two lift-off variations ........ 119
   6.4.1 Theoretical background and implementation ............................................ 119
   6.4.2 Verification of the inverse scheme using FEM ......................................... 121
6.5 Inverse scheme using LOIs based on measurement with magnetic field camera .... 123
   6.5.1 Theoretical background and implementation ............................................ 123
   6.5.2 Verification of the inverse scheme using measurement with magnetic field camera
   ......................................................................................................................... 125
      6.5.2.1 Experimental setup .............................................................................. 126
      6.5.2.2 Inverse process and discussions ............................................................ 129
6.6 Chapter summary ............................................................................................ 131

CHAPTER 7 CONCLUSION AND FURTHER WORK ................................................. 134
7.1 Concluding remarks ....................................................................................... 134
   7.1.1 ETREE modelling of SFEC and PEC inspections on multilayered structures .... 135
   7.1.2 Inverse schemes using LOI ................................................................. 137
7.2 Further work .................................................................................................... 138
   7.2.1 ETREE for EC forward problems involving natural and complex-shaped defects.. 138
   7.2.2 Extension of the inverse schemes with LOIs using sensor arrays ................. 139
   7.2.3 Magnetic field imaging using Magnetically Actuated Micromirrors .......... 140

LIST OF REFERENCES .............................................................................................. 142

APPENDICES ............................................................................................................. 155
   A. ETREE modelling code for SFEC inspection of multilayered structures .......... 155
      A.1 Main function ............................................................................................. 155
      A.2 Sub function ‘yy’ ........................................................................................ 156
      A.3 Sub function ‘matrixgen’ ............................................................................. 157
      A.4 Sub function ‘svd_equ’ ............................................................................... 159
   B. Establishment of the database used in inverse scheme ................................... 160
      B.1 Main function ............................................................................................. 160
      B.2 Sub function ‘dphidt’ ................................................................................... 162
      B.3 Sub function ‘findloi’ .................................................................................. 163

LIST OF PUBLICATIONS ......................................................................................... 164
   Journal papers ..................................................................................................... 164
   Conference papers .................................................................................................. 164
# LIST OF FIGURES

Figure 1. Structure of ENDE Techniques involved in the research ................................................................................. 9
Figure 2. Principle of electromagnetic induction and EC [12] ................................................................................................. 10
Figure 3. Typical MFEC excitation waveforms: (a) Multiplexed Injection mode (4 frequencies); (b) Simultaneous Injection mode (4 frequencies) [31] ................................................................................................. 11
Figure 4. (a) Principle of MFL inspection of pipeline; (b) MFL inspection system for rail track (SperryRail) [40] ...................................................................................................................................................... 13
Figure 5. Previous dynamic MFL inspection system .............................................................................................................. 14
Figure 6. (a) The field coupling paths within RFEC inspection; (b) the signals amplitude and phase vs. distance \( n \) times of pipe diameter, \( n \geq 0 \) from the exciter/driver coil [52] ......................................................................................................................... 16
Figure 7. Numerical methods mostly used in ENDE .............................................................................................................. 17
Figure 8. Analytical modelling for ENDE ........................................................................................................................... 20
Figure 9. General setups for ENDE experimental system .................................................................................................... 34
Figure 10. Schematic illustration of the research methodology ............................................................................................. 35
Figure 11. FE simulation for MFL inspection of flawed specimen ........................................................................................... 43
Figure 12. Contour plots of magnetic field in three independent axes over SF ........................................................................... 45
Figure 13. (a) The experimental setup for initial magnetisation of the rail sample; (b) close-up image of the crack in the sample; (c) schematic illustration of system setup for 3D field measurement of residual magnetic field ........................................................................................................................................... 47
Figure 14. Contour plots of the sensor x-axis, y-axis and z-axis magnetic field strength from the rail track sample, with the crack position superimposed on the contour plot ................................................................................................................. 49
Figure 15. Illustration of the source of the y-axis signal component the rail sample ................................................................. 49
Figure 16. 2D Simulation model for MFL under dynamic measurement .................................................................................. 51
Figure 17. Zoom-in views of distribution of eddy currents within the sample as the probe travels at the speed of (a) 10 m/s; (b) 30 m/s ......................................................................................................................................................... 52
Figure 18. Distribution of magnetic flux lines as the probe travels at the speed of (a) 10 m/s; (b) 30 m/s 53
Figure 19. Magnitude of \( B \) vs. X axis against probe velocity .................................................................................................. 54
Figure 20. Magnitude of \( B \) vs. X axis against depth of surface defect with the probe travelling at the speed of 30 m/s ............................................................................................................................................... 55
Figure 21. Proposed high-speed MFL inspection system with three sensor arrays ........................................................................... 56
Figure 22. Principle of the proposed high-speed ENDE inspection system ............................................................................ 57
Figure 23. The 2D axi-symmetric RFEC model set up in COMSOL .......................................................................................... 61
Figure 24. Flowchart of FEA simulating RFEC response to CSD ........................................................................................... 62
Figure 25. MVP in Log vs. the ratio of distance between excitation coil and pickup coil to OD of pipe at different frequencies ................................................................. 63
Figure 26. MVP in Log against the ratio of distance between excitation coil and pickup coil to OD of pipe at different excitation frequencies ................................................................. 64
Figure 27. (a) Amplitude and (b) Phase of MVP along base line at 40 Hz ......................................................... 65
Figure 28. (a) Schematic illustration of the excitation coil scanning over the CSD; (b) Magnitude of MVP vs. position of excitation coil; (c) Phase of MVP vs. position of excitation coil .............. 66
Figure 29. The relative sensitivity ($e$) of RFEC and EC vs. excitation frequency ............................................. 68
Figure 30. A cylindrical coil of rectangular cross-section above a three-layered conductor system .......... 74
Figure 31. A cylindrical coil of rectangular cross-section above a three-layered conductor system within a truncated region ........................................................................................................ 78
Figure 32. A 2D axi-symmetric eddy current model involving a cylindrical coil above a conductor with arbitrary number of layers, a pickup coil and a Hall sensor ....................................................... 79
Figure 33. The procedures of ETREE modelling for PEC ($\omega_0$ denotes the frequency harmonics within pulsed excitation current; $k$ denotes $k^{th}$ harmonic) ................................................................. 88
Figure 34. SFEC/PEC inspection of two stratified structures: (a) Two-layer structure (Structure 1); (b) Three-layer structure (Structure 2) .................................................................................. 89
Figure 35. Close view of a 2D axi-symmetric FE model for Structure 2 (areas in red represent air) .......... 90
Figure 36. Schematic experimental setup ........................................................................................................ 91
Figure 37. Driver coil with a Hall sensor ............................................................................................................ 91
Figure 38. Magnetic field vs. excitation frequency in logarithmic scale for (a) Structure 1 and (b) Structure 2 (markers represent actual data points) ............................................................... 93
Figure 39. Magnetic field per unit excitation current vs. excitation frequency in logarithmic scale for (a) Structure 1 and (b) Structure 2 (markers represent actual data points) ......................................... 94
Figure 40. Magnitude of PEC signals with respect to Air, Structure 1 and Structure 2 against time .......... 96
Figure 41. Magnitude of PEC differential signals with respect to Air, Structure 1 and Structure 2 against time ........................................................................................................................................ 96
Figure 42. A 2D axi-symmetric eddy current model involving a cylindrical coil, a conductive half-space and a Hall sensor ........................................................................................................ 104
Figure 43. A 2D axi-symmetric eddy current model with the Hall sensor placed in a distance from the axi-symmetric axis of the driver coil ................................................................................. 109
Figure 44. A 2D axi-symmetric eddy current model with the Hall sensor placed in a distance from the axi-symmetric axis of the driver coil (Top view) ......................................................................... 110
Figure 45. A 2D axi-symmetric eddy current model involving a cylindrical coil, a non-magnetic conductive half-space and a Hall sensor placed in a distance from the axi-symmetric axis of the driver coil ........................................................................................................ 112
Figure 46. The PEC excitation current ..................................................................................................................... 115
Figure 47. (a) The predicted magnetic field signals from the Hall sensor with respect to different lift-off increments; and (b) their first-order derivatives against time along with the zoom-in figure within time range from 0.13 ms to 0.15 ms ................................................................. 116

Figure 48. (a) LOI range width and (b) centre vs. Hall position ................................................................. 117

Figure 49. (a) LOI range width and (b) centre vs. conductivity of the sample ........................................... 118

Figure 50. Illustration of the inverse scheme using two lift-offs and one sensor ...................................... 121

Figure 51. Databases showing LOI vs. lift-off against conductivity for lift-off variation of (a) 0.5 mm and (b) 1 mm ........................................................................................................................................ 122

Figure 52. (a) Two subspaces of $(c_1, \sigma)$ for LOIs; (b) the difference between the two subspaces ....... 123

Figure 53. Illustration of the inverse scheme using two sensors and one lift-off ....................................... 125

Figure 54. ETREE model comprising of a driver coil, two Hall sensors and a conductive half-space .... 126

Figure 55. (a) The schematic illustration of PEC system; (b) The PEC probe used in the experiments .. 127

Figure 56. The measured and predicted excitation current ........................................................................... 128

Figure 57. The distribution of half of the coil field obtained using the magnetic field camera .............. 128

Figure 58. PEC signals from the two Hall elements via measurement and theory ..................................... 129

Figure 59. The PEC signals from the central Hall element and the offset Hall element with and without the lift-off variation ........................................................................................................ 130

Figure 60. The distribution of LOI vs. $\sigma$ and $z_1$ for the two Hall elements: (a) the centre Hall element; (b) the offset Hall element ............................................................................................................. 130

Figure 61. (a) The two subspaces $(z_1, \sigma)$ and (b) the difference between the two subspaces ............ 131
LIST OF TABLES

Table 1. Multiplexed injection MFEC VS. Simultaneous injection MFEC ........................................... 11
Table 2. Comparison of typical 2D analytical methods for ENDE ............................................................ 23
Table 3. Dimension and Material of the specimen .................................................................................... 44
Table 4. Dimension and properties of the excitation coil ......................................................................... 52
Table 5. Dimension and properties of the conductive specimen ............................................................... 52
Table 6. Dimension and properties of the defect ....................................................................................... 52
Table 7. Dimension and material of model ................................................................................................. 61
Table 8. Dimension and properties of each layers .................................................................................. 92
Table 9. Coil parameters ............................................................................................................................ 92
Table 10. Computation Time spent on ETREE and FEM for SFEC modelling ...................................... 95
Table 11. NRMSD between ETREE and experiment ............................................................................. 97
Table 12. NRMSD between TSFEM and experiment ............................................................................. 97
Table 13. Comparison of computing time for PEC modelling ................................................................. 98
Table 14. Parameters of the probe ........................................................................................................ 115
Table 15. Parameters of the driver coil .................................................................................................... 127
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SUMMARY

This thesis reports a comprehensive literature survey on Electromagnetic Non-destructive Evaluation (ENDE) and the investigation of forward problems and inverse problems of Sweep-frequency and Pulsed Eddy Current NDE using numerical and analytical modelling in collaboration with an experimental approach.

Firstly, Finite Element simulations have been conducted to (1) evaluate and assess the commercial Finite Element simulation packages, COMSOL and ANSOFT MAXWELL EM in terms of feasibility for simulation of ENDE forward problems including Eddy Current and Magnetic Flux Leakage; (2) apply Finite Element modelling as a comparative approach to verify the proposed analytical methods for forward models and inverse schemes.

In parallel with the numerical simulations, much more effort is put into implementation of efficient magnetic-field-based Eddy Current modelling, particularly for Sweep-frequency Eddy Current and Pulsed Eddy Current inspections on multilayered structures. The Extended Truncated Region Eigenfunction Expansion (ETREE) method has been proposed and verified. The formulation of closed-form expressions, which are in series of expansions of eigenfunctions, is useful to rapidly predict signals from solid-state magnetic field sensors rather than the traditional impedance signals measured using induction coils, as the dimension of sensors is taken into account in the model.

Based on ETREE models, the analytical expression of Lift-Off Intersection (LOI) occurring in eddy current inspection is formulated. This is adopted to investigate the characteristics of LOI and set up the database for inverse schemes. Following that, the inverse schemes for the estimation of lift-off of probes and conductivity of samples have been implemented by finding the measured LOI in the established database which comprises the relations of variable lift-off as well as conductivity with the corresponding LOI. The results from Finite Element simulations and experiments have proved the validity of the analytical modelling results as well as the proposed inverse schemes.
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tbody>
<tr>
<td>AE</td>
<td>Acoustic Emission</td>
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<tr>
<td>ACPD</td>
<td>Alternating Current Potential Drop</td>
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<tr>
<td>AMR</td>
<td>Anisotropic Magnetoresistive Sensor</td>
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<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
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<tr>
<td>CBMS</td>
<td>Computer-Based Modelling and Simulation</td>
</tr>
<tr>
<td>CSD</td>
<td>Circumferential Surface Defect</td>
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<tr>
<td>DIAS</td>
<td>Diagonal Section</td>
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<td>EC</td>
<td>Eddy Current</td>
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<tr>
<td>EFG</td>
<td>Element-free Galerkin Method</td>
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<tr>
<td>ENDE</td>
<td>Electromagnetic Non-destructive Evaluation</td>
</tr>
<tr>
<td>ETREE</td>
<td>Extended Truncated Region Eigenfunction Expansion</td>
</tr>
<tr>
<td>FBEM</td>
<td>Finite Boundary Element Method</td>
</tr>
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<td>FDM</td>
<td>Finite Difference Method</td>
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<tr>
<td>FE</td>
<td>Finite Element</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
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<tr>
<td>FEM</td>
<td>Finite Element Modelling</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>GMR</td>
<td>Giant Magnetoresistive Sensor</td>
</tr>
<tr>
<td>HORS</td>
<td>Horizontal Section</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IFT</td>
<td>Inverse Fourier Transform</td>
</tr>
<tr>
<td>ILT</td>
<td>Inverse Laplace Transform</td>
</tr>
<tr>
<td>IO</td>
<td>Input-Output</td>
</tr>
<tr>
<td>LOI</td>
<td>Lift-Off Intersection</td>
</tr>
<tr>
<td>MAE</td>
<td>Magneto-Acoustic Emission</td>
</tr>
<tr>
<td>MBN</td>
<td>Magnetic Barkhausen Noise</td>
</tr>
<tr>
<td>MEMs</td>
<td>Micro-electromechanical System</td>
</tr>
<tr>
<td>MFEC</td>
<td>Multi-frequency Eddy Current</td>
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</table>
MFL: Magnetic Flux Leakage
MPI: Magnetic Particle Inspection
MVP: Magnetic Vector Potential
MVPF: Magnetic Vector Potential Formulation
NDE: Non-destructive Evaluation
NDT&E: Non-destructive Testing & Evaluation
NRMSD: Normalised Root Mean Squared Deviation
OD: Outer Diameter
PDE: Partial Differential Equation
PEC: Pulsed Eddy Current
PERS: Perpendicular Section
PIG: Pipeline Inspection Gauge
PMF: Point Magnetic Field
PRF: Pulse Repetition Frequency
QNDE: Quantitative Non-destructive Evaluation
RFEC: Remote Field Eddy Current
SF: Surface defect
SFEC: Sweep-frequency Eddy Current
SOVP: Second-order Vector Potential
SPH: Smoothed Particle Hydrodynamic Method
SQUID: Superconducting Quantum Interference Device
SVD: Singular Value Decomposition
TREE: Truncated Region Eigenfunction Expansion
TSFEM: Time-Stepping Finite Element Modelling
UT: Ultrasonic Testing
VMF: Volume Magnetic Field
VRM: Variable Reluctance Method
CHAPTER 1
INTRODUCTION

This chapter presents an introduction to the research work, which has been conducted on ‘Theoretical and experimental investigation of ENDE for defect characterisation’. The achievements and problems existing in previous research are generalised and depicted as the background to ongoing study. This is followed by the aims and objectives of the research. The contributions of the current work are presented. Following that, the layout of this thesis and content in each chapter are summarised.

1.1 Research background

Electromagnetic Non-destructive Evaluation (ENDE) is widely applied to various engineering applications involving petrochemical, transportation, energy and nuclear industries. It comprises a number of techniques, which are based on Electromagnetism and electromagnetic field interaction with conductive specimens under inspection, such as Eddy Current (EC), Remote Field Eddy Current (RFEC), Magnetic Flux Leakage (MFL). They are also taken as complementary approaches to other NDE techniques such as Ultrasonic Testing (UT), Acoustic Emission (AE), Radiography, in an effort to non-invasively retrieve comprehensive information on the integrity of samples.

In order to gain knowledge about the functionality and performance of would-be ENDE inspection instruments or to optimise the designed probes, the physical phenomena particularly the underlying electromagnetic field and its response to variations inside conductive specimens under evaluation should be analysed quantitatively and comprehensively. To do this, ENDE systems can be experimentally evaluated and assessed using ENDE Benchmark problems. However, this approach is time-consuming and dependent on several critical conditions such as calibration samples, material properties and the size as well as shape of specimens. Compared with experimental approaches, computer-based modelling and simulation (CBMS), being the ‘economical’
approach is preferable and facilitates the analysis of electromagnetic phenomena taking place within ENDE systems. Numerical, analytical or semi-analytical methods are employed in CBMS to derive solutions to the governing equations, which are deduced from the well-established Maxwell’s Equations and reveal the relationship between parameters of ENDE systems and the resulting variations in electromagnetic quantities. The benefits from CBMS includes: (1) giving insight into electromagnetic phenomena in ENDE inspection systems; (2) providing hints in system design and optimisation; (3) to reveal the links of systematic parameters with the resulting field signals to solve the ENDE forward and inverse problems.

With the advent of application of solid-state magnetic field sensors to ENDE, two issues however have arisen ever since CBMS was taken as a complementary method to address the ENDE problems: the magnetic-field-based modelling and its efficiency in terms of simulation time and computation accuracy. Both of these issues play an essential role in modelling of current ENDE problems, especially relevant to utilisation of solid-state magnetic field sensors instead of traditional induction coils. Previous research has presented good agreement between simulation and experimental results for some ENDE problems, such as the impedance signal and its response to coated, flawed specimens using EC. Nonetheless, little attention has been paid to these two issues. Consequently, there is still much demand to implement efficient simulations with reference to magnetic-field-based ENDE inspections, which include for example, Pulsed Eddy Current (PEC), Sweep-frequency Eddy Current (SFEC), RFEC and MFL.

The research fulfils: (1) Magnetic-field-based Finite Element Analysis (FEA) for ENDE forward problems involving EC, RFEC and dynamic MFL inspections, through which the magnetic-field-based numerical simulations in 2D and 3D are investigated, and an adequate commercial package of FEA is selected for the study; (2) The realisation of efficient analytical modelling to predict magnetic field signals from solid-state magnetic field sensors used in SFEC and PEC; (3) The inverse schemes estimating lift-off of EC probes and conductivity of samples under PEC inspection using Lift-Off Intersection (LOI) in conjunction with the forward analytical model.
1.2 Aims and Objectives

1.2.1 Research aims

Based on the research background involving the issues in CBMS for ENDE problems, the aim of the research is to provide the forward analytical model for SFEC and PEC inspection of multilayered specimens with non-conductive and conductive layers, such as pipes with metallic coating and plane wings. This realises the fast and accurate prediction of magnetic field signals from solid-state magnetic field sensors. Following the forward modelling, the inverse schemes for obtaining the parameters of EC probes and specimens under evaluation are to be proposed. The research aims are detailed as follows:

- The evaluation of commercial FEA packages through a series of case studies regarding ENDE forward problems including MFL and RFEC;
- Realisation of fast and accurate simulations via analytical modelling for SFEC and PEC for the purpose of predicting the magnetic field signals from sensors instead of the traditional impedance signals from induction coils, and providing an analytical expression of LOI for SFEC and PEC, via which the characteristics of LOI and its dependence on the parameters of the inspection systems can be investigated;
- Implementation of inverse schemes to acquire the information regarding lift-off of EC probes and the conductivity of specimens under inspection, due to the demand for quantitative evaluation of the two parameters during the course of the inspection.

1.2.2 Research objectives

The objectives of the research are summarised as follows:

- To undertake a comprehensive literature survey of ENDE techniques involved in current research, i.e. SFEC, PEC, RFEC and MFL;
- To conduct magnetic-field-based Finite Element (FE) simulations of ENDE forward problems including RFEC inspection of metallic pipes with circumferential defects on its metallic coating, static MFL inspection of samples with irregular-shaped
cracks and dynamic MFL inspection of ferromagnetic specimens with surface cracks, via which the appropriate FEA software for simulations concerning SFEC and PEC inspection of multilayered structures is chosen to subsequently verify the analytical modelling and inverse schemes; The sub-objectives are listed as follows:

- To conduct case studies with regard to ENDE forward problems involving MFL and RFEC in order to evaluate the commercial FE simulation packages COMSOL and ANSOFT MAXWELL EM in terms of the feasibility and compatibility to the subsequent modelling for SFEC and PEC;
- To perform FE simulations using COMSOL for predicting magnetic field signals from magnetic field sensors during the course of SFEC and PEC inspections of multilayered structures, in an effort to provide comparative results from FE modelling to verify the analytical models and inverse schemes;
- To implement magnetic-field-based simulations via analytical modelling, in an effort to realise the fast and accurate prediction of magnetic field signals regarding SFEC and PEC inspections on multilayered structures; The sub-objectives are given below:
  - Fast and accurate analytical modelling using Extended Truncated Region Eigenfunction Expansion (ETREE) for predicting the magnetic field signals from sensors during the SFEC and PEC inspection of multilayered specimens;
  - To set up experimental SFEC and PEC inspection systems for the evaluation of multilayered specimens via magnetic field measurement, which involves design and implementation of inspection hardware and algorithms;
  - To verify the established analytical models for SFEC and PEC by comparing with experimental results and the FEA models in terms of computation time and calculation accuracy;
- To investigate the characteristics of LOI in SFEC and PEC through the proposed ETREE modelling, based on which the inverse schemes for estimating the lift-off of probes and the conductivity of samples are proposed;
  - To conduct the theoretical formulation for LOI taking place during SFEC and PEC inspection of conductive specimens with variable lift-off of the EC probe;
To propose inverse schemes for estimation of probe lift-off and the conductivity of the specimens under SFEC and PEC evaluation, based on the analytical modelling for SFEC and PEC;
To verify the proposed inverse schemes via FE simulations and practical measurement of magnetic field signals from a magnetic field camera.

1.3 Main achievements

Following the completion of the research objectives, the research outcome can be summarised as follows:

1. Literature survey that provides the state-of-the-art of ENDE techniques, simulations on ENDE forward problems, and inverse schemes for extracting the properties of the specimens under evaluation;
2. Evaluation and assessment of commercial FE simulation packages, i.e. COMSOL and ANSOFT MAXWELL EM through case studies of ENDE forward problems with reference to MFL and RFEC;
3. FEA of SFEC and PEC inspections on multilayered structures, conducted in COMSOL, in an effort to provide the comparative results of magnetic field signals from FE modelling for verification of analytical modelling and inverse schemes;
4. Magnetic-field-based analytical modelling, i.e. ETREE for predicting the magnetic field signals from magnetic field sensors during SFEC and PEC inspections on multilayered structures, which includes: (1) implementation of fast and accurate predictions of magnetic field signals in lieu of traditional impedance signals; (2) modelling which takes sensors’ dimension into account; (3) investigation of the characteristics of LOI in SFEC and PEC;
5. Inverse schemes for estimation of probe lift-offs and conductivity of a sample during the SFEC and PEC inspection, by using ETREE modelling to establish database of LOI in conjunction with interpolation functions;
6. Publication of papers in journals (IEEE, NDT&E International, etc) and presentation of the work to conferences (BINDT, WCNDT, etc), as listed in LIST OF PUBLICATIONS.

1.4 Thesis layout

Chapter 1 gives the outline of the thesis, which involves research background, aims and objectives, scope of work and general achievements related to the work.

Chapter 2 presents the literature survey of the state-of-the-art of ENDE techniques, in particular, EC along with SFEC as well as PEC, MFL and RFEC, numerical as well as analytical modelling for the design and development of ENDE. In addition, a review of the inverse process for quantitative ENDE is provided.

Chapter 3 presents the theoretical background of ENDE. It starts with a brief introduction to Maxwell’s governing equations, followed by a general overview of numerical solution to time-stepping as well as time-harmonic ENDE problems, and analytical solution to time-harmonic ENDE problems. Furthermore, the research methodology is presented.

Chapter 4 elaborates the case studies regarding static and transient MFL, and RFEC via FE simulations. This involves time-harmonic and time-stepping modelling. The two commercial FEA packages, COMSOL and ANSOFT MAXWELL EM are evaluated through the case studies.

Chapter 5 gives details about the theoretical and experimental investigations of SFEC and PEC inspections on multilayered structures. The analytical modelling using ETREE is implemented to predict the magnetic field signals from solid-state magnetic field sensors. The superiority of ETREE over FEA in terms of simulation time and computation accuracy is presented by comparing theory and experiment.
Chapter 6 focuses on the inverse schemes for lift-off estimation and conductivity evaluation using the ETREE model in conjunction with LOI. Following the derivation of the analytical expression of LOI in SFEC and PEC, the characteristics of LOI are investigated. Since it is found that LOI is closely related to the configurations of inspection systems and properties of the specimens, two inverse schemes are proposed based on the database established via ETREE modelling, and the acquired LOIs within the magnetic field signals from magnetic field sensors. These are verified using FE simulations and practical measurement with a magnetic field camera.

Chapter 7 summarises the research work, derives conclusions and outlines the further work, which is based on the current investigation.
CHAPTER 2

LITERATURE SURVEY

In this chapter, the ENDE techniques involved in the research are reviewed first. Their fundamental principles are briefly presented, followed by the state-of-the-art of their applications to ENDE inspection for various purposes. Theoretical studies of ENDE, which aim to explore the electromagnetic field underlying the inspection systems and predict the system responses to anomalies within specimens under evaluation, are elaborated. The current approaches of simulations for ENDE problems via both numerical and analytical means are surveyed critically. Afterwards the problems that need to be addressed in current research are identified.

2.1 ENDE techniques

ENDE is widely applied to engineering applications for evaluation of metallic structures, and is complementary to other NDE techniques such as Ultrasonics and Radiography [1]. ENDE techniques are based on the fundamentals of Electromagnetism and the interaction of electromagnetic fields with the materials under evaluation. They include EC, MFL, ACFM [2-6], RFEC, Magnetic Barkhausen Noise (MBN) [7], Magneto-Acoustic Emission (MAE) [8-9], Magnetic Particle Inspection (MPI) [10, 11], etc. This thesis will not discuss all ENDE techniques but focuses on the techniques involved in the current research, which can be categorised into two groups according to the location of the magnetic field used for inspection. Figure 1 presents the ENDE techniques associated with the research, which are put into the two groups: (1) near-field inspection methods including EC (MFEC and PEC) and MFL; (2) remote-field inspection methods such as RFEC.
2.1.1 Eddy Current

As shown in Figure 2, which presents the principle of EC, traditional EC employs an applied magnetic field, which is generated by an induction coil, namely a driver coil supplied with a sinusoidal current with a single frequency. The applied magnetic field (primary magnetic field) induces eddy currents in conductive specimens as the driver coil is deployed close to the specimen. Any anomalies within the specimen cause perturbation of eddy current. As a result, the eddy-current-induced magnetic field (secondary magnetic field), which opposes the applied magnetic field, changes in terms of the amplitude and phase. Consequently, the variation in magnitude and phase of the net magnetic field (superposition of primary and secondary magnetic fields) can be found via measurement by pickup coils or solid-state magnetic field sensors/sensor arrays. Through intensive analysis of the measured signals, the anomalies are detected, identified and quantitatively evaluated [12-14].
Since EC was discovered and introduced in ENDE, it has become the preferable technique for the inspection of non-magnetic specimens such as aluminium plate. However, since the penetration depth of the eddy current is inversely proportional to the square root of the excitation frequency, the conductivity and permeability of a conductor [1], traditional EC exhibits the drawback including the limited depth information of specimens due to the fixed penetration depth of the eddy current during the inspection of a specific conductor.

In a bid to overcome the pitfalls of traditional EC and improve inspection efficiency, detection capability, evaluation feasibility and compatibility, etc [15-20], some methods have been proposed, which are established based on traditional EC. So far, MFEC [21-23] and PEC [24-29] are the common advanced EC techniques showing better performance in ENDE based on the eddy current phenomena. Both MFEC and PEC employ a range of frequencies in excitation. MFEC adopts multi-frequency sinusoidal excitation ranging from several Hertz (or DC) to Mega Hertz. The excitation at each frequency is generated sequentially or simultaneously, and lasts for a specific time within which the signal processing is carried out. In contrast, in PEC a transient excitation current in the form of a pulsed, rectangular or step waveform is introduced. Its frequency band is wider than that of a sinusoidal excitation with a single frequency. Both techniques take advantage of the fact that the integrity of specimens at different depths can be assessed in one single excitation process, in light of which both MFEC
and PEC have exhibited outstanding merits in the inspection of coated pipelines, specimens with subsurface defects and multilayered structures such as aircraft wings, etc [25].

Nowadays, inspection systems using MFEC and PEC are commercially available and play an important role in the evaluation of pipelines and aircraft especially. However, it is hard to tell which technique is superior. For MFEC, two modes of excitation are used in industrial applications: multiplexed injection [30] namely SFEC, and simultaneous injection [31]. Typical excitation waveforms of these two modes are presented in Figure 3. A comparison of the two operational modes of MFEC is shown in Table 1 [31].

![Figure 3. Typical MFEC excitation waveforms: (a) Multiplexed Injection mode (4 frequencies); (b) Simultaneous Injection mode (4 frequencies) [31]](image)

<table>
<thead>
<tr>
<th>Table 1. Multiplexed injection MFEC VS. Simultaneous injection MFEC</th>
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<tr>
<td><strong>System Construction</strong></td>
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<tr>
<td>Low cost; only one system needs to be constructed, which can provide excitation at several frequencies in discrete time.</td>
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<tr>
<td><strong>Signal analysis and feature extraction</strong></td>
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<td><strong>Time consuming during Inspection</strong></td>
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<td><strong>Applications in industry</strong></td>
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In a sense of excitation mode, PEC can be taken as a specific type of simultaneous-injection MFEC, since the harmonics of various frequencies within PEC excitation current are fed into the driver coils simultaneously. Whereas, the difference between PEC and simultaneous-injection MFEC lies in the allocation of the drive power. Via spectrum analysis, PEC allocates the power over the entire frequency harmonics of the excitation current, and the power amplitude at each harmonic is inversely proportional to the frequency [28, 29]. In contrast, the power at each harmonic for simultaneous-injection MFEC does not vary with the frequency. Therefore, PEC is superior to the typical simultaneous-injection MFEC on its low-cost instrumentation for wideband inspection and high signal-to-noise ratio especially for detection of subsurface defects [32]. It also shows the advantage over SFEC on higher inspection efficiency and feasibility for dynamic evaluation.

It is noteworthy that so far, the research on signal interpretation and the inverse process of MFEC inspection are based on coil impedance signals plotted in the complex plane instead of the magnetic signals measured by solid-state magnetic field sensors or sensor arrays, and multilayered specimens have not attracted much attention. Therefore, not only the magnetic field signals of MFEC as well as PEC and their responses to multilayered specimens, but also inverse models for real-time pattern recognition and characterisation, need intensive investigation.

2.1.2 Magnetic Flux Leakage

Compared to EC, MFL is a magnetisation-based NDE technique for detection and characterisation of anomalies or inclusions such as corrosion, cracks, grooves, etc. using a powerful magnet (permanent magnet or DC electromagnet) deployed inside a Pipeline Inspection Gauge (PIG) [33-35]. Figure 4(a) illustrates the principle of MFL inspection of pipelines, when the probe encounters a reduction in wall thickness, e.g. corrosion or an abrupt discontinuity, such as a crack. The applied magnetic field leaks outside the specimen around the anomaly area and the leakage field is detected with either
Hall-effect sensors/sensor arrays or induction pickup coils. This method is suitable for the detection and characterisation of defects in ferrous materials [37-39].

Figure 4. (a) Principle of MFL inspection of pipeline; (b) MFL inspection system for rail track (SperryRail) [40]

Equipment using MFL has been constructed especially for the dynamic inspection of rail tracks and pipelines. For MFL inspection of pipelines, PIGs are driven by electric motors and run inside the pipeline. Since the permanent magnets or electromagnets are aligned with the axial line as well as the diameter of pipeline, the inspection system can detect circumferential and axial defects in pipelines at a speed up to 4 m/s [41-43].

The dynamic MFL inspection system developed by Michigan State University College is schematically presented Figure 5 [43]. There are three sensors deployed at individual positions for magnetic field measurement: (1) Sensor #1 for magnetic flux leakage measurement; (2) Sensor #2 for remote field eddy current measurement, which takes place in a distance from the excitation i.e. the magnetiser; (3) Sensor #3 for measurement of magnetic flux linkage, which indicates the lift-off between magnetiser and the specimen surface, and is implemented by obtaining the coil reluctance during inspection i.e. Variable Reluctance Method (VRM). Although the system is applicable to measurement at speeds less than 10 m/s, it is not suitable for high-speed inspection of rail track and oil pipeline. Since the magnetiser travels at a high speed such as 30 m/s, the penetration depth of the eddy currents and magnetic field, which depends on the probe velocity and the electrical properties of the sample, is less than the thickness of
rail tracks or pipeline due to the skin effect analogous to EC with high frequency excitation, which hampers the application of the remote field eddy current to the field measurement under high-speed movement. Secondly, although VRM can be employed for compensation of the variation in lift-off, it is actually the measurement of the average effect of magnetic flux linkage, which is incapable of locating, sizing or characterising defects. Moreover, the profile of magnetic field distribution will be shifted due to high-speed motion of the MFL probe. It is therefore impracticable to apply a single sensor for capturing the defect signals.

![Figure 5. Previous dynamic MFL inspection system](image)

As exhibited in Figure 4(b), other kinds of MFL inspection systems are installed on train wheels in order to obtain the structural information of rail tracks. They have shown advantages over the other techniques in non-contact implementation, dynamic inspection at speeds up to 10 m/s, high sensitivity to transverse and slanted cracks with different depth (up to 15 mm) and angle (up to 25° from the surface of the rail track) [44, 45].

However, so far the MFL evaluation on rail tracks in particular is not applied to in-service inspection, which indicates that the inspection of the rail track cannot commence unless there is no transportation travelling on it. This causes an inconvenience as railway lines have to be closed for inspection, and increases the possibility of derailment due to lack of inspection. In order to overcome the disadvantages, it is demanded that the MFL inspection system is set up on trains running for public transportation, at a speed of about 50 m/s and used periodically for the evaluation of rail track. Before constructing a high-speed ENDE inspection system,
intensive investigation should be conducted on the characteristics of the magnetic field, and magnetic field response to defects in rail track, which are inspected at high speed.

2.1.3 Remote Field Eddy Current

RFEC traditionally uses low frequency excitation in order to make the electromagnetic field penetrate through the metallic body of the structure, and implement in the inspection of the ferromagnetic and non-magnetic wall of pipes. This technique has approximately equal sensitivity to defects on the inner or the outer pipe wall [46].

The principle of RFEC and the signal amplitude and phase against the length of the pipe are illustrated in Figure 6 [47]. The applied magnetic field is generated by the coaxial driver coil which is excited with a either a sinusoidal [48, 49] or pulsed waveform [50, 51] current. The detectors, which are pickup coils or magnetic field sensors, are deployed near the inner pipe wall and with a distance of nearly two pipe diameters from the driver coil. The electromagnetic wave is transmitted through two coupling paths, between the detector and the driver coil, which has been shown via simulations: direct coupling path in direct coupling zone and indirect coupling path in remote field zone [52-56]. The magnetic field through direct coupling path distributes inside the pipe and is subject to rapid attenuation due to eddy current induced within the pipe wall. The field via indirect coupling path penetrates through the pipe wall with less attenuation, and travels outside the pipe. The two fields are coupled in the transition zone where the field through indirect coupling path diffuses back into the pipe. In the remote field region, the strength of the field due to indirect coupling is higher than the field inside, which indicates the integrity of the pipe wall.
Most research has been conducted via theoretical and experimental investigations in a bid to (1) give better understanding of the RFEC phenomenon and the acquired signals [57, 58]; (2) optimise systems in terms of the deployment of the detector and selection of the optimal excitation frequency [59-61]; (3) implement FEA in 2D and 3D and analytical approaches to facilitate the simulations of RFEC in various circumstances [62-67]. Nevertheless, the research is more focused on the RFEC inspection of pipes without metallic coatings while the corrosion and defects in the metallic coating of the pipe have become a pressing concern recently. Therefore, the investigation of RFEC inspection of defects in metallic coatings and pipe walls is demanded, regarding the applicable system configuration.

2.2 Numerical Simulations of ENDE

Numerical simulation methods (as listed in Figure 7) for solving electromagnetic problems governed by Partial Differential Equations (PDEs) can be categorised into two groups, according to the mesh conditions: mesh-based and meshless methods. For the group of mesh-based methods, Finite Element Modelling (FEM) [68] originating from Finite Difference Method (FDM) [69] is dominant in the solution of sophisticated electromagnetic problems, followed by Finite Boundary Element Method (FBEM) [70] and Hybrid Methods, which integrate FEM with FBEM [71, 72]. In the group of
meshless methods, Element-free Galerkin Method (EFG) is the most frequently used method in ENDE [73, 74], which is based on Smoothed Particle Hydrodynamic Method (SPH) [73] common since the 1990s.

Figure 7. Numerical methods mostly used in ENDE

FEM is preferred to other numerical methods in light of its flexibility in the description of the problem, capability of coping with nonlinear magnetic materials without too many restrictions, fast solutions to specific electromagnetic problems, compatibility in dealing with problems concerning multi-physics e.g. electromagnetic issues coupling with heat transmission [68]. Since extensive studies in the computation and implementation of FE simulations have been conducted, the FE simulations have been realised by using not only self-designed programs in laboratories for specific purposes but also commercial simulation packages that can solve canonical problems as well as practical issues defined by users. For example, the commercial packages such as COMSOL [75], ANSOFT MAXWELL EM [76], Infolytica MagNet [77], facilitate modelling and simulation by embedding state-of-the-art FEM solvers, easy-to-use graphical interfaces, automatic as well as optimised meshing and computing algorithms and multiple post-processing functions for result display and analysis.

Along with the rapid development of computers, FE simulation has been shifting from 2D modelling to 3D modelling which is more suitable to handle asymmetric problems
that cannot be simplified to a 2D problem using azimuthal coordinates, for instance [78]. 3D FE simulation is beneficial to theoretical study of ENDE, because the geometries of specimens modelled in simulations are usually asymmetric [79]. The commercial simulation packages which are prevalent in electromagnetic modelling are capable of implementing simulations in not only 2D but also 3D. As a result, we can obtain a better understanding of magnetic fields underlying the inspection systems and to verify the experimental results.

Unfortunately, although commercial packages as well as self-designed programs have versatility in handling different electromagnetic problems, they have limitations and drawbacks in dealing with specific issues concerning ENDE. These include laborious post-processing to derive the signal features frequently focused on in ENDE and inappropriate solvers or mesh algorithms for some typical ENDE issues. Consequently, the simulation study, especially the research on FEM for ENDE, has not halted, but still attracted much attention from researchers in ENDE. The study is focused on: (1) Study on the feasibility as well as adaptation of commercial packages and their application in ENDE; (2) Research on suitable solvers and algorithms for ENDE modelling in 2D and 3D.

The application of numerical simulations to ENDE problems started with FDM, which was proposed by Dodd and Deeds. The equation governing eddy current properties was solved with a high accuracy [12]. However, since FDM has the limitation in dealing with complicated geometries, it was replaced by FEM, which is the dominant method for investigation in ENDE. Dai et al. applied a hybrid FEM and FDM computational model for studying PEC [80]. In contrast, FEM has been used independently for simulations of ENDE such as EC and MFL [42, 81-85]. The FE simulation models have been developed for 2D and 3D. FEM solvers for electromagnetic calculations have been expanded from magnetostatic and electrostatic solvers to transient solvers. All of these present best performance of FEM for ENDE simulations.
Bowler firstly introduced FBEM for investigating EC [12]. Chen et al. used FBEM for a thin-opening crack in a plate conductor [70]. With consideration of the pitfalls of FBEM, the hybrid FEM-FBEM method has been proposed, which couples FEM with FBEM. Klimpke proposed a hybrid FEM-FBEM solver for investigation of magnetic field problems [71]. Sabariego and Gyselinck adopted a hybrid FEM-BEM model for 2D time-harmonic eddy current problems, which is accelerated with the Fast Multipole Method (FMM) [72]. In addition, the Meshless Method is a new approach in numerical methods and it has shown usefulness in some areas related to electromagnetic field. Xuan et al. introduced this method in EC using the breakthrough in EFG for Static and Quasi-Static electromagnetic field computation [73, 74]. In FE simulations of dynamic MFL inspection, 2D and 3D FE simulations have been conducted by adopting magnetic vector potential formulation (MVPF) in conjunction with edge-based FEM by Michigan State University College [42], based on which the prototype dynamic MFL inspection system for pipelines has been constructed. However, the inspection speed under investigation was less than 10 m/s, which needs to be improved for high-speed inspection.

Even though much research work has been focused on FE simulations for ENDE including EC, MFL, etc, the drawback of FEM is noted. Compared with analytical modelling particularly for EC, FE simulations are mesh-dependent and subsequently time-consuming. Besides, tradeoffs between modelling accuracy and computing power usually need to be made, which would result in low-efficiency simulations.

### 2.3 Analytical Modelling of ENDE

Analytical modelling provides closed-form solutions written in compact equations to differential equations that are derived from Maxwell’s Equations, or equations derived from equivalent models based on physical phenomena. This approach has shown the merits of fast computation without loss of computation accuracy in the simulation of electric motors, magnetic brakes, etc.
As illustrated in Figure 8, the approaches in analytical modelling for ENDE can be classified into two categories according to the theories underlying the electromagnetic phenomena. The first approach is based on the electromagnetic induction between coils used in inspection systems and the conductive specimens. A model in the form of an equivalent circuit is built up, which is analogous to the circuit usually adopted in the analysis of transformers, since the electromagnetic phenomena in ENDE especially EC inspection is identical to that in transformers. The interaction of magnetic field with defects in conductive specimens is represented by mutual inductance in the equivalent circuit, which varies with the self-inductance of a single filament of excitation coil and specimens. The impedance signal frequently used in ENDE is derived from the integral of the closed-form impedance of the filament over the cross-section of the coil. This approach has been implemented in the commercial analytical package, CIVA that is specifically designed for ENDE [86].

Figure 8. Analytical modelling for ENDE

The other approach is based on the Maxwell’s equations, which govern the electromagnetic phenomena. Since there are differential equations derived from Maxwell’s equations, which govern and predict the electromagnetic field distribution as well as its variations in ENDE inspection systems. The closed-form solutions to the unknowns in the equations via this approach bring about the macroscopic description of
electromagnetic field within ENDE systems. The application of this approach to ENDE began with Dodd and Deeds’ analytical modelling of EC inspection of a conductive half-space and a 2-layered plate as well as rod with probe-coil using integral expressions [87, 88]. The governing equation was formulated with azimuthal coordinates and solved with the Separation of Variables method. The first-order and second-order boundary conditions were taken into account for calculating the coefficients in the expression of the solution to the magnetic vector potential in each subdomain. Since the solution has Bessel functions of the first order, the Fourier-Bessel equation was used to simplify the integration derived from boundary conditions. With appropriate formulation of the governing equations and boundary conditions, the simulated signal of coil impedance shows good agreement with experimental results.

Following Dodd and Deeds’ analytical modelling, the 3D analytical modelling for asymmetric EC problems due to coils with arbitrary shapes, inclusions in conductive half-space, wobbles during pipeline inspection, etc have been studied. Since the conventional Separation of Variables method is not suitable for 3D formulation, specific decoupling methods, especially for magnetic vector potential in governing equations have been researched and employed in the 3D formulation of ENDE problems. Also the superposition approach is used for solving the problem of a filamentary wire above a flawed half-space [89] and first-order Born approximation is used in impedance calculation of an arbitrary EC probe [90]. In addition to these, the formulation using second-order vector potential (SOVP) has been widely used in the analytical modelling of EC issues due to its inherent advantage that the solution can be derived from the solution of the respective scalar ones [91-93].

However, Dodd and Deeds’ modelling suffers from the redundant calculation of infinite integrals, since the problem domain has an infinite width. In light of this, Theodoulidis has proposed Truncated Region Eigenfunction Expansion (TREE) [94, 95]. With TREE, the domain of interest is truncated and recast into a region with finite dimensions, a consequence of which, the solution with integration of infinity is changed into finite eigenfunction expansions. Although additional boundary conditions are imposed on the
surface/line of truncation, the calculation of series expansions and extra boundary conditions does not take much computing power and is faster compared to the integral calculation in previous modelling.

Furthermore, since truncation of the modelling domain results in an approximation of the unbounded region of problem, TREE has made it likely to establish the analytical simulation of an ENDE problem, in which the conductive specimen has finite dimensions. Until now, TREE has provided accurate analytical solutions to ENDE problems such as an EC probe scanning over multilayered rods [96, 97], wedges [98], flawed plate [99], etc. The modelled signal of the impedance change due to the variable positions of an EC probe moving above the specimen under investigation has shown good agreement with measured signals. It has advantages over numerical simulations in providing fast and accurate solutions, which benefits the inverse process of ENDE problems, for the acquisition of information on the integrity of the specimen from obtained signals, and has made TREE a milestone in solving ENDE problems with an analytical approach.

Until now, many of the analytical packages for ENDE modelling and simulation have not been widely commercialised, apart from CIVA that has been used in analytical simulations, particularly the modelling of EC in industry. Nonetheless, some self-designed software e.g. TEDDY [100] and MPEC5 [101] has exhibited the advantages of analytical modelling over numerical simulation and been prevalent in the research of ENDE especially EC, MFEC and PEC. These two packages excel in providing the predicted impedance signals due to multilayered specimens.

For 2D modelling for EC inspection of multilayered specimens, transformer-based modelling, Dodd and Deeds’ modelling and TREE modelling are applicable. In general, TREE modelling exhibits the superiority over the other two approaches in terms of providing accurate and fast prediction of impedance signals from pickup coils. The comparison of the three methods is detailed and shown in Table 2.
### Table 2. Comparison of typical 2D analytical methods for ENDE

<table>
<thead>
<tr>
<th>Methods</th>
<th>Transformer-based</th>
<th>Dodd-and-Deeds based methods</th>
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<tr>
<td></td>
<td></td>
<td>Integral expression</td>
</tr>
<tr>
<td><strong>Merits</strong></td>
<td></td>
<td>Feasible to simulations of MFEC and PEC inspections on conductive half-space as well as multilayered specimens.</td>
</tr>
<tr>
<td></td>
<td>Simplification of inspection system using transformer model, which has the fastest computation speed.</td>
<td></td>
</tr>
<tr>
<td><strong>Drawbacks</strong></td>
<td>The application is limited to simple simulations of variation of coil impedance due to flawed/unflawed conductive specimens. The computation suffers the infinite integral expressions.</td>
<td>The closed-form solutions comprise Bessel functions, trigonometric functions and exponential functions in infinite integral expressions. The model can only be carried out with azimuthal formulation.</td>
</tr>
<tr>
<td><strong>Featured software</strong></td>
<td>CIVA</td>
<td>TEDDY and MPEC5</td>
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It should be pointed out that most analytical methods are devoted to predicting the impedance signals from pickup coils instead of magnetic field signals from solid-state magnetic field sensors, which are employed in modern ENDE inspection. It is imperative to implement magnetic-field-based analytical modelling for ENDE particularly for SFEC and PEC.

### 2.4 Quantitative ENDE and inverse process

Quantitative NDE (QNDE) for ENDE aims to quantitatively analyse and identify defect types, location and growth from the measured electromagnetic signals, as well as their extracted features. With the help of theoretical and experimental study involving signal processing for QNDE, the inverse modelling process, which plays a vital role in QNDE, facilitates the establishment of relationships between signal response, signal features, material variations and defect characteristics.
The ENDE forward modelling process essentially includes prediction of interaction of electromagnetic field and corresponding signal variations due to anomalies within specimens via theoretical and experimental study. Compared to this, the ENDE inverse process plays a vital role in extracting a wealth of information about specimens and defect characterisation from features found in signals, which give best representation of characteristics of defects. This results in the explicit reconstruction of defect profiles under interrogation [102].

The inverse models of QNDE have been intensively investigated for several years focusing on EC. The approaches can be categorised in two groups according to the methodologies: Theoretical model and Black-box model. Theoretical models are mostly based on numerical modelling, e.g. FEM and analytical modelling such as Dodd and Deeds’ method have been employed in the EC inverse process. The measured signals are compared with the signals predicted by the theoretical models, which are built from an initial guessed value, in terms of the dimensions and properties of defect as well as the specimen. This inverse process is iterative. The iteration will not stop until the minimum error between measured value and predicted value is reached. The approach relies on three major factors: efficiency of a theoretical model; initial guess towards solution; algorithms for seeking minimum during iteration. For the theoretical model, fast FEM [22] and analytical models [103] are used, in addition to which, least squares method [104, 105], annealing method [103], tabu search method [106] and genetic algorithms [107] have been used in searching for a global minimum among calculated values while avoiding the localised minimum that results in computational error.

The Black-box model used in the inverse process of ENDE is mostly referred to the artificial neural network (ANN) [108]. ANNs have been widely used in the fields of Engineering, Medical and Chemical applications in light of the fact that: (1) ANNs can be made applicable to various areas after getting trained with the known information; (2) ANNs are highly adaptive to the ‘inputs’, and has very good robustness with respect to the ‘outputs’; (3) In the wake of rapid development in computer-based computation,
ANNs are readily implemented in conjunction with various algorithms for fast computation as well as accurate characterisation and estimation.

Generally speaking, an ANN is a mathematical model [109]. Multiple equations depicting the relation between the inputs and the outputs are included within the models, most of which are written in linear correlation. The coefficients, namely neurons, of the equations vary with different inputs whilst they are defined and specified within one particular application. It is imperative to derive these coefficients to formulate the equation and establish the implicit relations which are mostly intricate to be obtained via traditional mathematical means such as inverse approaches for linear algebraic equations, parameterization of the formulations, etc. ‘Training’ is adopted to acquire these coefficients in conjunction with the data already known, which is distinctive compared to other computing algorithms. By using training, the distribution (or patterns) of the known data can be found and expressed in a mathematical way along with the neurons adaptive with the input data. It should be noted that the computational cost relevant to ANN usually concerns the training of the ANN. Algorithms are thereby developed regarding the training and employed to minimise the deviation of the estimated data against the true values, which is broadly taken as the criterions for inverse process, optimization or statistical approximation. The mostly used algorithms include the evolutionary approach, simulated annealing, expectation-maximisation and non-parametric methods [110]. Special care is also taken with regard to different applications that dominate the characteristics of the input-output (IO) relations, and determine whether the correlations are written in either linear or non-linear expressions.

In the inverse process for ENDE, the ENDE inspection systems are parameterised with neurons in the ANN trained by the predicted or experimental signals, which involve either coil impedance signals or magnetic field signals. The ANN is established after every neuron and its corresponding transfer function are derived from training, because of which this approach costs more time especially in training the ANN. Until now, ANNs have exhibited the merits in mapping and reconstructing the stress corrosion by inversely analysing the distribution of conductivity of specimens [111-114]. However, it
is noted that before deriving the ANN for particular ENDE inverse process, the laborious training is demanded, which needs a large amount of data acquired from either experiments or simulations. Besides, the characterisation accuracy of an ANN is highly dependent on the size of the dataset input during the training process, which may cause low accuracy in the inverse process if the dataset is not large enough. The two facts mentioned above could hinder the implementation of fast inverse process and low identification accuracy.

2.5 Summary and problems identified

An extensive literature survey on ENDE techniques especially advanced inspection methods i.e. MFEC, PEC, RFEC, dynamic MFL and ACFM is reported in this chapter. Followed by a brief summary of numerical and analytical methods and their simulations employed in theoretical study of ENDE to investigate the magnetic field distribution within the inspection systems and its responses to anomalies in specimens under evaluation.

From the literature survey, there are several problems of ENDE inspection needing to be addressed especially for online and real-time evaluation on in-situ specimens, fast forward simulation models and the inverse process based on efficient models, which involve the following:

- Fast forward solvers using analytical methods for SFEC and PEC have been researched for several decades, based on the integral expression proposed by Dodd and Deeds. Nevertheless, the traditional analytical modelling approaches are implemented to predict the impedance signals or potential drop across induction coils. In consideration of the fact that more solid-state magnetic field sensors are used in inspection systems, the magnetic-field-based analytical modelling, particularly for simulation of magnetic field signals from magnetic field sensors/sensor arrays needs to be implemented.

- Although FEA is well established particularly for electromagnetic simulations, the FEM for ENDE problems still needs to be investigated in order to realise the
magnetic-field-based ENDE simulations and verify the proposed analytical method.

- Even though commercial FE simulation packages have been commercially available, the selection of an appropriate package for SFEC and PEC simulations is demanded. This is fulfilled by using FEM implemented in COMSOL and ANSOFT MAXWELL EM to simulate ENDE problems involving MFL and RFEC from static to transient analysis. The studies give an insight into the electromagnetic phenomena within the systems, and provide a comprehensive understanding and direction in the design as well as the optimisation of the inspection systems.

- The lift-off of EC probes and the conductivity of samples are unknown, and need to be evaluated during SFEC and PEC inspections on the specimens with complex surface geometry or variable thickness of the non-metallic coating. This is regarded as an inverse EC problem. Even though ANN has been found advantageous for some inverse ENDE problems from previous research, it is inapplicable to this particular inverse problem. It is because ANN is a training-based method, which needs a large number of calibrated samples for better accuracy, and thus the training process would be intricate and time-consuming. Therefore, the inverse scheme, particularly for estimating the lift-off of the EC probe and conductivity of specimen needs to be proposed with the help of recent theoretical modelling, whilst the fast computation and high estimation accuracy are realised.
CHAPTER 3
THEORETICAL BACKGROUND OF ENDE

In this chapter, the theoretical background of ENDE which applies to not only numerical and analytical modelling of ENDE but also the experimental approach is discussed. The fundamental groundwork is exhibited, based on which the research methodology to address the problems identified in Chapter 2 is presented and has been adopted in the comparative research.

3.1 Maxwell’s equations and deduced governing equations for ENDE

Since ENDE employs an electromagnetic field in all its techniques and applications, it obeys the rules of Electromagnetism governed by a series of physical laws, which comprise Maxwell-Ampere’s Law, Faraday’s Law and Gauss’ Law in electric and magnetic forms [75, 115]. The equations depict the laws in either integral form or differential form and mathematically describe the electromagnetic relationship between the electric field and magnetic field within a system, which are the foundation of Electromagnetism. Suppose all the materials within the system are linear and homogeneous, for time-variant fields, the Maxwell’s equations are [115]:

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (1) \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2) \]

\[ \begin{cases} \nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \\ \nabla \cdot \mathbf{D} = \rho \\ \nabla \cdot \mathbf{B} = 0 \end{cases} \quad (3) \]
where, $\mathbf{H}$ and $\mathbf{B}$ denote the magnetic field intensity and magnetic flux density respectively; $\mathbf{E}$ stands for the electric field intensity; $\mathbf{J}$ and $\mathbf{D}$ are the current density and electric displacement current, respectively; $\rho$ is the electric charge density; $t$ represents time.

The electromagnetic phenomena in a closed system can be investigated on a macroscopic level as long as the Maxwell’s equations are solved mathematically in conjunction with appropriate boundary conditions and constitutive relationships representing the properties of each material, which are shown as follows [115]:

\[
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}
\]

(4)

\[
\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})
\]

(5)

\[
\mathbf{J} = \sigma \mathbf{E}
\]

(6)

where, $\mathbf{P}$ denotes the electric polarisation vector; $\mathbf{M}$ is the magnetisation vector; $\varepsilon_0$ represents the permittivity of vacuum; $\mu_0$ and $\sigma$ stand for the permeability of vacuum and the electrical conductivity, respectively.

For most ENDE problems with a frequency of up to several megahertz, since the wavelength of the electromagnetic wave is much larger than the dimension of a system, the displacement current $\mathbf{D}$ vanishes. Thus, Eqs (1) and (3) can be simplified to [115]:

\[
\nabla \times \mathbf{H} = \mathbf{J}
\]

(7)

\[
\begin{cases}
\nabla \cdot \mathbf{B} = 0 \\
\nabla \cdot \mathbf{J} = 0
\end{cases}
\]

(8)
With the introduction of the magnetic vector potential $\vec{A}$, the convolution of which gives $\vec{B}$:

$$\vec{B} = \nabla \times \vec{A} \tag{9}$$

The electric field can be expressed as:

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \tag{10}$$

where, $V$ denotes the electric potential, which is scalar. Note that $\vec{A}$ satisfies the Coulomb Gauge:

$$\nabla \cdot \vec{A} = 0 \tag{11}$$

In consideration of Eqs. (4) and (5), substitute Eq. (9) into (11), the governing equations describing the electromagnetic field in ENDE systems are deduced from Maxwell’s equations. For time-harmonic fields in particular, the governing equation is [75]:

$$\sigma j\omega \vec{A} + \nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) - \sigma \vec{v} \times (\nabla \times \vec{A}) = \vec{J}_s \tag{12}$$

where, $\mu$ is the material permeability; $\sigma (j\omega \vec{A}) = \vec{J}_e$ denotes the eddy current density; $\vec{J}_s$ denotes the source current density; $\vec{v}$ denotes the media velocity.

For time-variant fields, Eq. (12) changes into [75]:

$$\sigma \frac{\partial \vec{A}}{\partial t} + \nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) - \sigma \vec{v} \times (\nabla \times \vec{A}) = \vec{J}_s \tag{13}$$
Eqs. (12) and (13) are the general governing equations in time-harmonic form and time-variant forms for ENDE respectively, which apply to EC and MFL for example. It is apparent that Eqs. (12) and (13) are partial differential equations that can be solved using numerical or analytical methods. After \( \vec{A} \) is derived by solving Eqs. (12) and (13), all the electromagnetic entities in an ENDE system are obtained, and accordingly the underlying electromagnetic field is analysed.

3.2 Numerical approach in solving time-harmonic and time-stepping problems of ENDE

Numerical approaches are based on the application of iteration methods with the prerequisite that the ENDE problem domain of interest is discretised into a number of elements which make up a mesh. In conjunction with the shape function of each element, governing equations of ENDE are substituted with interpolation functions set up in all elements. The value on each node of element is iterated in consideration of the boundary conditions, which imply the initialised values. The iteration does not halt until the residual is less than the error tolerance.

Numerical approaches to solving time-harmonic and time-stepping problems of ENDE have significant advantages over analytical methods in dealing with specimens with complex-shaped defects or arbitrary surfaces and the ENDE problem with nonlinear magnetic materials, due to better description of problem geometries and properties by means of discretisation. However, numerical methods mostly rely on the conditions of elements and mesh. In other words, by using a better mesh leading to high convergence of the simulation results, a better computational accuracy is achieved with of the disadvantage of a longer computing time. This intrinsic pitfall hampers the wider application of numerical approaches to not only ENDE forward problem but also ENDE inverse problem.
3.3 Analytical approach in solving time-harmonic problems of ENDE

The analytical approach aims to solve the closed-form solution to the unknown i.e. $\vec{A}$ in Eqs. (12) and (13). It begins with the separation of variables method because $\vec{A}$ depends on the other two orthogonal components. Following the separation of these two components, two independent differential equations are obtained, whose general solutions comprise Bessel functions of the first and second order, associated Bessel function, trigonometric function or exponential functions along with several coefficients to be determined using boundary conditions. The Fourier-Bessel transform method and Bessel-orthogonality method have been employed to simplify the equations of boundary conditions, which show the continuity of the electromagnetic field over boundaries. Once the coefficients are calculated, the closed-form solution to ENDE problem is solved.

It should be noted that no mesh is needed in the analytical approach. The coefficients in the closed-form solution derived from the analytical approach are independent of some factors such as frequency and locations of probes, which upset the numerical approach in re-meshing the problem region of interest and deriving precise mathematical expressions of solution. In light of this, the analytical approach has been presenting high-speed and high-accuracy computation in modelling for ENDE. Nevertheless, not all ENDE problems can be solved with the analytical approach. Problems with nonlinear magnetic materials, perplexed surface or defect are usually difficult using the analytical approach. For instance, a specimen with a crack enforces additional boundary conditions that result in non-linear equations to be computed, rarely with algebraic methods but numerical iterations. Consequently, the efficiency is influenced by the iterations for approximating the exact solutions to non-linear equations.

3.4 Experimental investigation of ENDE

The theoretical background still applies to the experimental investigation of ENDE since the Electromagnetism underlying the inspection systems is governed by
Maxwell’s equations and the deduced expressions, and consequently the measured signals can be verified and analysed via comparison with the predicted signals. The effective interpretation and analysis of measured signals can be conducted along with the investigation of electromagnetic phenomena that are consistent with theoretical background.

Dedicated experimental setups play a vital role in the experimental study of ENDE. Basically, the setup comprises excitation sources for generating the electromagnetic field injected into the specimen under evaluation or an inherent source i.e. residual magnetic field within specimens due to their construction process, sensing modules for measuring and quantifying the electromagnetic field, the signal conditioning and processing module that provides automatic and systematic results of evaluation by using sophisticated techniques in hardware and algorithms in software, power sources. The schematic figure of general system setups for ENDE is presented in Figure 9.

The sensing module is of great importance in all the components of the experimental setups, since it has a close connection with the measurement and quantification of the electromagnetic field for ENDE. Accordingly, the verification as well as interpretation of experimental results, validity of systematic approaches, and feasibility of inspection techniques considerably relies on the efficiency of this module.
Until now the application of induction coils for measuring the magnetic field has been restrained. Firstly, coils actually quantify the rate of change in magnetic field instead of the magnetic field intensity. Secondly, some extraneous factors have strong influence on the characteristics of coils. For example, the coil sensitivity is highly dependent of the excitation frequency. The uniformity of coil windings and homogeneity of coil wire have effect on the coil inductance and reactance, which are related to the coil resonance frequency and the operation bandwidth.

Thanks to the development in semi-conductors and the discovery of electromagnetic material variation due to external magnetic field, the solid-state magnetic field sensor and sensor array have been employed in ENDE experiments using advanced material assessment techniques, which include Hall sensors [116-119], Giant Magneto resistive (GMR) sensors [119-121], Anisotropic Magneto resistive (AMR) sensors [121] and Superconducting Quantum Interference Device (SQUID) [17]. The application of the solid-state magnetic field sensors and sensor arrays, not only simplify the experimental setups and shortens the time for system construction, but also enhances the performance of systems and facilitates the analysis of the experimental results.
3.5 Research methodology

In order to address the problems of ENDE presented in Chapter 2, based on the theoretical background of ENDE in terms of numerical and analytical approaches, and experimental method, a series of studies on theoretical and experimental investigation of ENDE have been conducted. As shown in Figure 10, the work covers: (1) Evaluation and assessment of commercial FEM packages via a series of case studies concerning FE simulations for MFL and RFEC; (2) realisation of magnetic-field-based analytical modelling for SFEC and PEC inspection of stratified conductive specimens in a bid to implement fast and accurate simulations for SFEC and PEC, which facilitates the inverse process; (3) verification of the proposed analytical modelling method via FE simulations conducted in selected software and experiments; (4) the inverse scheme for estimating lift-off of the EC probes and conductivity of samples, based on the characteristics of LOI via analytical modelling, which is verified via FEA and experiments.

Figure 10. Schematic illustration of the research methodology
3.5.1 Theoretical study

3.5.1.1 FEA for EC, dynamic MFL and RFEC

FE simulation has exhibited advantages in modelling ENDE problems concerning complicated structures and material (nonlinear magnetic and inhomogeneous), components having mechanical movement involving velocity coupled with displacement. In order to verify the proposed magnetic-field-based analytical method, a series of case studies have been conducted, which concern time-harmonic FEM and time-stepping FE simulations for ENDE. Commercial FE simulation packages, COMSOL and ANSOFT Maxwell EM have been employed. The evaluation of these two packages in magnetic-field-based modelling for the ENDE problems is conducted via FEA for MFL and RFEC. The general introductions to these two packages are presented as follows [75, 76]:

- ANSOFT Maxwell EM is a commercial package for numerical simulation of frequency and time domain electromagnetic fields in complex structures. It implements FEM whilst allowing BEM codes and has strongly-coupled electromagnetic, drive circuit and mechanical formulations. It also integrates several numerical modules for solving specific problems, such as electrostatic, magnetostatic, quasi-static and transient problems as well as those involving eddy currents.

- COMSOL is a Multiphysics Modelling Package for numerical simulation of the physical process, which can be described using partial differential equations (PDEs). State-of-the-art solvers are embedded in the package for the user to choose, to address complicated problems quickly and accurately. It contains specific modules for different applications, which involve Chemical Engineering, Earth Science, Heat Transfer, Micro-electromechanical systems (MEMS), Structural Mechanics and Electromagnetics. The electromagnetic module is applied to electromagnetic field simulations from static and quasi-static to microwaves and photonics. It also realises the field analysis in the static,
transient and frequency domain using static FEM solver, Time-Stepping FEM and Time-Harmonic solver, respectively.

After the selection of an FE simulation package, the forward problem of SFEC/PEC inspection of multilayered conductive structures has been investigated via FEA implemented in COMSOL in an effort to provide the numerical results, for comparison between numerical simulation and analytical modelling. Two FEM solvers have been employed to compute the models, which are set up for the simulation of time-harmonic and transient fields. As for SFEC, the parallel computation of FE simulations, within the range of excitation frequencies has been developed in Simulink in conjunction with COMSOL, which dramatically saves the computational time. In contrast, during the course of FE simulations for PEC, the measured excitation current through experiment in lieu of the predicted current is used in order to mitigate the discrepancy between the practical current and the predicted current, which would aggravate the deviation of simulation results from experiment.

3.5.1.2 ETREE modelling for SFEC and PEC

FE simulations are found time-consuming in computation due to their high dependency on the mesh, although they have shown versatility in handling rigorous conditions. Tradeoffs between computation time and accuracy have to be made, which hinders the application of FEA in efficient ENDE simulations.

An analytical approach, namely ETREE modelling, is proposed to implement fast and accurate simulations, particularly for SFEC and PEC inspections on stratified conductors, simulating the metallic structures of aircraft. Compared to the integral expression and previous TREE methods, ETREE modelling is applied in predicting the magnetic field signals from solid-state magnetic field sensors, rather than the impedance signals from induction coils. The calculation time is reduced due to (1) the replacement of infinite integrals with a series of eigenfunction expansions; (2) most coefficients
being frequency-independent. In addition, the computational accuracy/convergence can be more readily controlled by choosing an adequate number of expansions.

Following the separation of variables method, which is used in the integral expressions and TREE methods, the magnetic field intensity at a particular point within the problem domain in 2D axi-symmetric coordinate system is expressed. Based on the deduced equations, the closed-form expression of the magnetic field in a volume, defined by the dimension of the sensor can be easily formulated by introducing the volume integral.

The ETREE modelling for PEC becomes straightforward after the formulation of magnetic field signals from magnetic field sensors is established for SFEC. The inverse Fourier Transform is used to derive the temporal PEC response to the samples from its spectrum in frequency domain. The fact that the most coefficients in the expression for magnetic field signals for SFEC are frequency-independent, and the calculation of inverse Fast Fourier Transform is rapid, contributes to the implementation of fast and accurate ETREE modelling for transient field, especially for PEC.

Following the derivation of the analytical expressions of magnetic field signals for SFEC and PEC, the closed-form equations of LOI found in SFEC and PEC signals when the probe lift-off varies can be formulated. Through this formulation, the characteristics of LOI and its dependency on the configuration of inspection systems become explicit, which benefits in the establishment of the database depicting the relationship of LOI with parameters of the systems involving lift-off of the probe and conductivity of the sample under evaluation. The investigation develops the inverse schemes proposed for inverse estimation of the conductivity of the metallic specimen and the lift-off of the probe in conjunction with the ETREE forward modelling and the characteristics of LOI.
3.5.2 Experimental study

The experimental study mostly focuses on the implementation of measurement system for SFEC and PEC to acquire the magnetic field signals from the sensors, and evaluation of stratified conductors.

The system consists of an EC probe with a driver coil and magnetic field sensor/sensor array, power amplifier, stratified conductors, a multi-channel data acquisition card and computer. The driver coil is supplied with current of either sinusoidal or rectangular waveform to generate the applied magnetic field as a function of frequency or time. The Hall sensors and AMR sensors are used to acquire the magnetic field signals for SFEC/PEC and 3D magnetic field measurement. The measured signals are amplified by the instrument amplifier and recorded via the multi-channel data acquisition card installed in a computer. The software interface is designed in LabVIEW to perform the signal display and data pre-analysis. The intensive analysis of the measured signals is performed with MATLAB.

During the course of the experiments, both the magnetic field signals and the excitation current are obtained. The current signals are employed in the forward SFEC and PEC simulation to obtain the predicted magnetic field signals, whilst the measured field signals are utilised to verify the proposed models, particularly the ETREE model, to highlight its advantage over FEM in terms of fast computation and high accuracy.

In addition, the practical magnetic field signals picked up from two Hall elements of a magnetic field camera are adopted within the inverse scheme in an attempt to verify the proposed inverse scheme in the evaluation of sample conductivity as well as the estimation of EC probe lift-off.
3.6 Chapter summary

Based on the aims and objectives of the research, following a brief introduction of Maxwell’s equations that govern Electromagnetism and the governing equations derived from them for ENDE, the general approaches: numerical and analytical modelling for deriving the solutions towards the governing equations and the experimental approaches for ENDE are summarised. The methodology for the research on theoretical and experimental study for ENDE is presented along with specific approaches employed for achieving the objectives of the research.

Following this chapter, Chapters 4 to 6 report: (1) the case study in the FE simulations for MFL and RFEC in an effort to select the appropriate simulation package which provides FEA results for verification of ETREE modelling and inverse schemes in reference to SFEC and PEC; (2) the fast magnetic-field-based analytical modelling (ETREE) for SFEC/PEC inspection of multilayered structures; (3) the investigation of LOI and its dependency on parameters of EC systems; (4) the inverse schemes using LOI to derive lift-off of EC probes and conductivity of samples, and the verification of the inverse schemes via FE simulations and experiments.
CHAPTER 4
FINITE ELEMENT ANALYSIS FOR ENDE

In order to gain knowledge of Electromagnetism, and verify the analytical modelling for SFEC and PEC via FEA, this chapter concentrates on the FE simulations realised by using two commercial packages i.e. COMSOL and ANSOFT MAXWELL EM, with respect to ENDE problems involving (1) 3D magnetic field sensing in MFL inspection of samples with irregular-shaped cracks via static analysis; (2) dynamic MFL inspection of flawed samples via transient analysis; and (3) RFEC inspection of flawed coating of pipes via time-harmonic analysis. The advantage and disadvantage of each package is evaluated by using individual software for addressing the following issues: (1) 3D FEM; (2) FE simulations for Electromagnetics in conjunction with mechanical translation of components within models; (3) the modelling of the remote field concerning the large-dimension components within models. The outcome of the studies includes: (1) the evaluation of the two commercial FE simulation packages for ENDE problems; (2) the implementation of magnetic-field-based FEM for ENDE, which provides the FEA results for verification of the proposed analytical model of SFEC/PEC and inverse schemes.

4.1 Case study I: FEA for MFL

4.1.1 FE simulations for MFL with irregular-shaped crack

The case study focuses on the evaluation of COMSOL in 3D FE simulations for ENDE forward problems and its compatibility in magnetostatic analysis.
4.1.1.1 Background

As the magnetic field in MFL systems complies with the well-established Maxwell’s equations, which govern Electromagnetism, FEA has been employed in solving the Maxwell’s equations that apply to MFL magnetic fields in a bid to unveil the underlying electromagnetic phenomena, and characterise the magnetic field variations due to the occurrence of surface defects within the specimens under investigation. Therefore, FEA is beneficial not only to verify an experimental study but also to provide models for defect characterisation in MFL systems.

Because MFL systems mostly use electromagnets with DC excitation or permanent magnets with high magnetisation, the electromagnetic field in MFL systems can be taken as a static field. Because of this, the analysis of the magnetic field has been conducted in electromagnetostatic mode. Consequently, the time-variant Maxwell’s equations in either integral or differential form can be simplified since all time-dependent terms vanish.

Suppose the materials concerned in the model are isotropic, linear and homogeneous. The static magnetic phenomena in MFL are governed by the simplified Maxwell’s equation along with one constitutive relationship while the time-dependent terms vanish, as follows [75]:

\[
\begin{align*}
\nabla \times \overline{H} &= 0 \\
\overline{B} &= \mu_0 \mu_r \overline{H}
\end{align*}
\]  
(14)

Note that the field is static, therefore, the magnetic field intensity \( \overline{H} \) can be written by introducing the magnetic field scalar potential \( v_m \) as:

\[
\overline{H} = -\nabla v_m
\]  
(15)
The magnetic quantities can be computed as long as the unknown i.e. $v_m$ in Eq. (15) can be derived. To fulfil that, numerical approach for getting the solution to Eq. (15) was chosen. With consideration to the establishment and definition of MFL models, FEA was adopted for analysis into MFL for characterisation of an irregular defect. COMSOL was selected to implement modelling and simulation of MFL in 3D.

4.1.1.2 Simulation setup

As shown in Figure 11(a), a surface defect (SF) was introduced in an arbitrarily shaped magnetic specimen to be interrogated. The close view of the SF is exhibited in Figure 11(b). The shape of the SF is an irregular ‘Γ’ shape, which is used to simulate a typical natural crack in the rail head. The width of the slot is 2 mm while the depth is 5 mm. The angle between the horizontal section (HORS) and perpendicular section (PERS) of the SF is 90°. There is also a diagonal section (DIAS) between HORS and PERS with a 135° angle. The SF is located at the edge of the specimen, and in particular, PERS is positioned in the localised area with a curved surface, which brings about a formidable challenge in identifying the shape and orientation of SFs. The information on dimensions and material of the specimens is listed in Table 3.

![Figure 11](image1.png)

(a) Simulation model (inset: top view of the flawed area)

(b) Illustration of the irregular-shaped surface defect

Figure 11. FE simulation for MFL inspection of flawed specimen
### Table 3. Dimension and Material of the specimen

<table>
<thead>
<tr>
<th>Magnetic Specimen</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Relative Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>70</td>
<td>70</td>
<td>35</td>
<td>250</td>
</tr>
</tbody>
</table>

The simulation model of the magnetic specimen with the SF was built in COMSOL with 3D coordinates in order to implement a better description of the problem. The 3D magnetostatic linear solver was employed to calculate $\nu_m$, following which the distribution of the magnetic field over the SF in three independent axes, namely $B_x, B_y, B_z$ were obtained and investigated. Since the FE 3D simulation takes a long computing time, some assumptions are made, as follows:

- The applied magnetic field generated by the permanent magnets which are not included in the models is uniform and homogenous in the defect area with its flux direction along $x$ axis (Simulation of traditional MFL);
- The characteristics of the residual magnetic field within the specimen after pre-magnetisation are assumed to be analogous to those of the applied magnetic field, in terms of flux direction along $x$ axis and field homogeneity (Simulation of MFL measuring residual field without applied field);
- The magnetisation of the permanent magnets or residual magnetisation of the specimen is 1 T and it is aligned along the length of the specimens;

In order to get better results in 3D, a fine mesh was generated within the problem region of interest. Instead of hexagonal elements, the 3D model was discretised into a number of tetrahedrons. In addition, the mesh quality was enhanced around the SF in order to achieve accurate magnetic field distribution without too much sacrifice in computing time.
4.1.1.3 Simulation results and experiment

Distributions of the magnetic field in three axes i.e. $B_x, B_y, B_z$ were obtained above the specimen at a 1 mm distance, and are individually illustrated in Figure 12. The results from superposition of $B_x, B_y, B_z$ are also presented in Figure 12. Note that the superposition of the three components of the magnetic field was implemented by using the equation:

$$B_s = \left( \frac{B_x + B_y + B_z}{B_{xm} B_{ym} B_{zm}} \right)$$

(16)

where, $B_s$ is the superposition of the three components. $B_{xm}$, $B_{ym}$ and $B_{zm}$ denote the maximum values in the plots of $B_x, B_y$ and $B_z$, respectively.

Figure 12. Contour plots of magnetic field in three independent axes over SF

Since the SF perturbs the lines of flux of the magnetic field through the specimen, and causes some of the flux lines to leak outside the specimen. The leakage magnetic field acquired over the surface of the specimen indicates not only the location of the SF but also its dimensional information including shape and orientation, which can be seen from Figure 12.
Unfortunately, a comprehensive ‘image’ of the defect is not delivered by analysis of the distribution of a single field component i.e. $B_x$ or $B_z$. The contours of $B_x$ and $B_z$ are incapable of indicating DIAS and PERS while giving a strong indication of the shape and orientation of HORS. The shape and orientation of the SF cannot be completely identified even though the integration of data from $B_x$ and $B_z$ is employed in signal processing, which is commonly used in traditional MFL inspection. The reason for the drawbacks is that HORS of the SF considerably perturbs the distribution of $B_x$ and $B_z$ since it is perpendicular to the flux lines of the applied magnetic field, which results in a good indication of HORS. In contrast, DIAS and PERS are partially-aligned and fully-aligned with the applied field respectively, as a result of which it is difficult to determine the shape and orientation of DIAS as well as PERS by analysis of $B_x$ and $B_z$.

Analysis of the field components measured in traditional MFL inspection systems shows that there is a distinct lack of information for the characterisation of arbitrarily shaped defects. From the FEA results, especially the contour of $B_y$, it can be seen that the shapes and orientations of DIAS and PERS of the SF are identified, which shows the effectiveness of defect characterisation by investigating the results of 3D magnetic field measurement simultaneously. As shown in Figure 12, by integration and fusion of the signals from $B_x$, $B_y$ and $B_z$, the ‘image’ of the SF is acquired, and especially the shape and orientation of each section of the SF are clearly identified.

In summary, the findings from FEA simulation explore the potential applications of 3D magnetic field measurement to enhance the detectability, and characterisation of defects with irregular shapes, in addition to traditional measurement of $B_x$ or $B_z$, which has not caught much attention in previous work.
A series of experiments was also conducted to present the merits of 3D field measurement in MFL inspection of magnetic samples with cracks. A section of rail track with a natural crack was used in the investigation. In order to implement 3D residual magnetic field measurement, the sample was magnetised using permanent magnets before the test, and the residual magnetisation in the samples was used to investigate the flux leakage around the defects with the magnets removed. The experimental configuration for initial magnetisation of the sample and the image of the crack are shown in Figure 13(a) and Figure 13(b), respectively. Figure 13(c) shows the schematic illustration of system setup for 3D field measurement of residual magnetic field. As can be seen in Figure 13(b), the natural, irregularly shaped, surface crack takes place at the area close to the corner of the rail head.

![Figure 13](image.png)

Figure 13. (a) The experimental setup for initial magnetisation of the rail sample; (b) close-up image of the crack in the sample; (c) schematic illustration of system setup for 3D field measurement of residual magnetic field
The magnetic field sensor used in the experiments was a HMC1023 3-axis anisotropic AMR sensor. The sensor incorporates three separate AMR Wheatstone Bridge sensor elements, one for each axis of sensitivity. These sensor elements are maximally sensitive to magnetic fields, aligned parallel their axes of sensitivity, with the output for an applied field being proportional to the cosine of the angle between the field line and the axis of sensitivity. As illustrated in Figure 13(c), the sensor was placed over the surface of the rail head and interfaced to a 4-channel data acquisition card through high gain signal processing electronics, to allow simultaneous data acquisition from all three-sensor axes. The software interface was developed in LabVIEW, and MATLAB was used for data processing and result plotting. Readings were taken with the sensor positioned at 1 mm intervals in a grid around the defects.

Figure 14 shows the output of the three sensor axes for the tests on the section of rail track. It can be seen from the region with highest values in the plot of $B_x$ that $B_x$ gives an indication of the position of PERS of the crack, but relatively poor positional information about DIAS of the crack. This is due to the diversion of the magnetic field around the crack in this area. However, there is a trough representing the minimum values in the $B_y$ plot that directly corresponds to DIAS. It is also noted that the measurement results of $B_z$, which is usually used in field measurement fails to deliver the implication about the crack shape.
Figure 14. Contour plots of the sensor x-axis, y-axis and z-axis magnetic field strength from the rail track sample, with the crack position superimposed on the contour plot.

The reason why $B_y$ gives indicative information on DIAS of the crack can be explained by reference to Figure 15. On PERS of the defect, the field lines directly cut across the crack, causing the magnetic field to leak into the air, with very little displacement in the y-direction. However, in DIAS of the crack the field lines flow around the defect causing minimal leakage in the x-direction and the z-direction but a substantial displacement of the field in the y-direction.

Figure 15. Illustration of the source of the y-axis signal component the rail sample.
4.1.1.4 Summary of static FEA for MFL

The FEA for MFL reveals the magnetic field over the defect region, in particular the distribution of \( B_y \) in the immediate area of the irregularly-shaped defect, which shows that measurement of the y-axis field component not only complements the x-axis and z-axis signals but also enhances the detection and characterisation of defects in specimens. The simulation study also indicates the necessity of 3D magnetic field measurement for characterisation of irregular-shaped defects especially in terms of shape and orientation during MFL inspection.

Following the FEA, a test was undertaken for MFL inspection of specimens with a natural irregularly shaped crack. The magnetic field was quantified using a 3-axis AMR field sensor. It is found that the y-axis sensor gives a predictable output containing signal features that clearly correspond to the defect position. Although the test results are not as significant as the results from FEM, they exhibit a good correlation between defect position and sensor output. \( B_y \) is particularly useful in detecting the diagonally orientated section of the crack, whereas \( B_z \) give very little indication of crack position. Although there are localised discrepancies in the magnetic field distribution between experimental and simulated results, due to the inhomogeneous field and nonlinear magnetic material of the arbitrary shaped test samples, the overall distribution of the 3D magnetic field from experimental study has proved that the 3D field measurement is essential for detection and identification of irregular-shaped defects involving shape and orientation, which is proposed via the simulation study. As a result, the use of a three-axis system would be advantageous in certain situations to give orientation information, especially where irregularly shaped defects or defects orientated close to parallel to the applied field are expected.

The FEA is conducted with COMSOL in lieu of ANSOFT MAXWELL EM. It has been found that the setup of 3D FE models in ANSOFT MAXWELL EM is intricate, and the utilisation of customised mesh condition is hardly carried out. Furthermore, it is readily implemented to model the uniform magnetic field cross the specimen by imposing
appropriate conditions on boundaries in COMSOL while in ANSOFT MAXWELL EM this modelling approach is inapplicable.

### 4.1.2 FE simulations for dynamic MFL inspection

The simulation on the dynamic MFL inspection system involves (1) the assessment of ANSOFT MAXWELL EM for FE simulations for ENDE forward problems concerning the inspection probes moving above the specimens; (2) the investigation of eddy currents due to the movement of the probe and the transient magnetic field distribution under the influence from the translating probe; (3) the characterisation of MFL signals with varying defect properties.

#### 4.1.2.1 Simulation setup

As illustrated in Figure 16, a 2D simulation model built up in ANSOFT MAXWELL EM represents the cross-section of the moving MFL probe and the steel specimen. The distributions of magnetic flux lines and eddy currents in the specimen were observed, and the magnetic field leakage was measured with probe velocities varying from 0 m/s to 30 m/s with variations in defect depth.

![Figure 16. 2D Simulation model for MFL under dynamic measurement](image)

The geometric parameters of the probe, specimen and surface defects are listed in Table 4, Table 5 and Table 6. The lift-off between the probe and the specimen is kept at 1 mm while the standoff between the sensor array and the specimen is 0.5 mm.
### Table 4. Dimension and properties of the excitation coil

<table>
<thead>
<tr>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
<th>Turns</th>
<th>Cross-section Shape</th>
<th>Material</th>
<th>Current source (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
<td>400</td>
<td>Rectangular</td>
<td>Copper</td>
<td>10*</td>
</tr>
</tbody>
</table>

* DC excitation. The eddy currents in the coil are neglected.

### Table 5. Dimension and properties of the conductive specimen

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Thickness (mm)</th>
<th>Cross-section Shape</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>8</td>
<td>Rectangular</td>
<td>Steel (Conductivity=2e6 S/m, Initial relative permeability=50, Maximum relative permeability=100)</td>
</tr>
</tbody>
</table>

### Table 6. Dimension and properties of the defect

<table>
<thead>
<tr>
<th>Width (mm)</th>
<th>Depth (mm)</th>
<th>Cross-section Shape</th>
<th>Material</th>
<th>Flaw type</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4, 5, 6, 7, 8**</td>
<td>Rectangular</td>
<td>Air slot (Relative Permeability=1.0, conductivity=0 S/m)</td>
<td>Surface artificial defect</td>
</tr>
</tbody>
</table>

** Through-wall defect

4.1.2.2 Simulation results and discussion

In conventional static MFL inspection systems with DC excitation, there is no current in conductive specimens. In contrast, eddy currents are generated within the specimen when dynamic MFL inspection systems are employed. By using the simulation software, the distributions of eddy currents at velocities of 10 m/s and 30 m/s are illustrated in Figure 17(a) and Figure 17(b), respectively.

![Figure 17](image-url)

**Figure 17. Zoom-in views of distribution of eddy currents within the sample as the probe travels at the speed of (a) 10 m/s; (b) 30 m/s**
Compared to the case where the velocity of the probe is 0 m/s and thus there is no eddy current in the specimen, eddy currents exist within the specimen when the MFL probe has relative speed to the steel specimen, even though the excitation current is DC. Moreover, the eddy current generated by the probe in motion is distributed behind the legs of the ferrite core, and the profile of eddy currents is dependent on the probe speed. As a result, as illustrated in Figure 17, when the probe speed is increased, more eddy currents concentrate on the specimen surface and longitudinally stretch a greater distance after the probe. It is understandable that such a skin effect is also applied in dynamic MFL inspection. Consequently, it is practicable to arrange electromagnetic sensor arrays, which consist of multiple sensing elements, behind the probe to measure the magnetic field for defect detection indicated by eddy currents in this region.

In each case, the magnetic flux lines representing the magnetic field within the system are presented in Figure 18(a) and Figure 18(b).

![Figure 18](image)

**Figure 18.** Distribution of magnetic flux lines as the probe travels at the speed of (a) 10 m/s; (b) 30 m/s

From Figure 18, it can be seen that the distribution of magnetic flux lines is relatively sparse under the right leg of the ferrite core whilst it is dense under the left leg of the core. It is also noticeable that most flux lines concentrate in the regions behind each leg of ferrite core. In contrast to the case where the probe is static and the magnetic field distribution is symmetric with respect to the slot, the profile of the magnetic field is distorted in dynamic MFL because of the eddy currents generated in the specimen due to high-speed movement of the probe and is asymmetric with respect to the rectangular
slot defect. Moreover, the distortion of the magnetic field is directly proportional to the probe velocity.

The intensities and features of MFL signals from the dynamic MFL inspection were also investigated. The analysis of MFL signals directs not only the selection of the magnetic sensor (array) based on specifications such as sensitivity, bandwidth, and measuring range but also the signal-feature-based defect characterisation.

The magnitudes of the horizontal component of the magnetic flux leakage ($B_x$) between two legs of the ferrite core against varying probe velocities are shown in Figure 19.

![Figure 19. Magnitude of $B_x$ vs. X axis against probe velocity](image)

From the Figure 19, it can be found that:

- Compared to the velocity case of 0 m/s, the shape of the magnetic field in each high-speed case is asymmetric. It is noteworthy that the asymmetric signals from high-speed MFL are analogous to those for angular defects using static MFL. The two peaks of the MFL signal occur around the edges of the slot defect. The
difference in amplitudes of the two peak values also reflects the direction of movement of the probe. Moreover, the magnitude of the differences between the two peak values increases with increased probe speed;

- In general, the magnetic field strength decreases when the probe speed is increased. Therefore, eddy currents due to probe movement not only distort the profile of the magnetic field but also decrease the intensity of the magnetic field.

From the simulations above, it can be seen that the defect width can still be determined from the MFL signal by choosing the signal peaks as features. Subsequently, the relationship between the defect depth and the MFL signal of the dynamic MFL inspection system was observed by conducting simulations with various depths of surface defect whilst keeping the probe velocity constant at 30 m/s. The defect depths used in the simulations are 4, 5, 6, 7 and 8 mm, where the 8 mm defect is a through-wall defect. The results are presented in Figure 20.

Figure 20. Magnitude of $B_x$ vs. X axis against depth of surface defect with the probe travelling at the speed of 30 m/s
From Figure 20, it can be seen that, similar to the static case, because more field leaks outside the specimen in the flawed region with increased defect depth, the magnitude of the MFL signal in a high-speed MFL inspection system is directly proportional to the depth of surface defect. In addition, there is distortion in MFL signals. Consequently, the defect characterisation of its depth should be integrated with the speed at which the probe moves and the direction in which it travels.

### 4.1.2.3 Proposed high-speed ENDE inspection system

With the consideration of the incapability of the previous systems use in high-speed measurement of up to 30 m/s, based on the simulations and analysis, a high-speed MFL inspection system for defect detection and characterisation in pipeline and rail track is proposed. According to the simulations, it is difficult to use a single magnetic sensor for capturing the profile of the magnetic field distribution. Therefore, magnetic sensor arrays are exploited, which are essentially a group of sensing elements placed in 1D/2D grid. Their deployment is determined by the investigation of the relationship between the measured magnetic field and the inspected defect. As illustrated in Figure 21, three sensor arrays are adopted individually for standard MFL, residual magnetic field measurement and eddy-current-induced magnetic field measurement. The functionality of the system is shown in Figure 23.

![Figure 21. Proposed high-speed MFL inspection system with three sensor arrays](image-url)
The sensor array for standard MFL distribution i.e. Sensor Array 1, which is deployed between the legs of the ferrite core, is employed for detection of defects and inclusions within the specimen after the specimen is magnetised. Nonetheless, since the proposed system runs at high speed, the level of magnetisation is lower than that of static MFL system. In the simulation, to saturate the steel specimen, the intensity of the applied magnetic field should be around 2 T, which is determined by obtaining the field strength corresponding to the minimum permeability of the specimen. However, the magnetic field on the outer surface of the specimen due to the MFL probe movement is only 0.9 T, which indicates that the specimen is not magnetised to saturation. Therefore, some defects located in the region may be undetectable or incorrectly characterised. To overcome this disadvantage, residual magnetic field measurement is also adopted simultaneously. Residual magnetic field measurement, as a low-magnetic-field measurement, has attracted much attention in ENDE. By using the method, not only defects but also stress within the specimens can be evaluated [35, 37]. With respect to the system, the introduction of residual magnetic field measurement avails the “second interrogation” of the defect as well as the investigation of the stress. As a result, Sensor
Array 3 is deployed before the magnetisation device to obtain the distribution of residual magnetic field.

In addition to MFL and residual magnetic field measurement, a sensor array for the eddy-current-induced magnetic field is also applied in the proposed system. Since the sensitivity of the detection is dependent on the density of eddy currents circumferentially flowing in the specimen, the sensor array needs to be deployed in the proximity of the moving probe. Therefore, in the proposed system, Sensor Array 2 is placed behind the left leg of the ferrite core to capture the field distribution during the inspection at high speed.

As are implied in the numerical simulations, the three sensor arrays should be high-sensitivity and high-bandwidth and the sampling frequency for signal acquisition should be high enough with respect to online high-speed measurement. By integration of the three signals from the three sensor arrays, defects in conductive specimens can be detected and characterised. The implementation of the proposed system and the experimental investigation will be involved in the future work.

**4.1.2.4 Summary of FEA for dynamic MFL**

An intensive theoretical study of a high-speed ENDE inspection system using MFL excitation via Time-stepping Finite Element modelling (TSFEM) implemented in ANSOFT MAXWELL EM is presented in this chapter. From the preliminary research, it can be found as follows:

- The previous dynamic MFL inspection system using techniques of MFL, RFEC, and VRM along with induction coils and single magnetic field sensor is not feasible or compatible for inspection at high speed due to the restriction of application for RFEC and VRM and the formidable challenge of the acquisition of the magnetic field.
- The high-speed movement distorts the electromagnetic field within the system and under-saturates the ferromagnetic specimens, which brings about difficulties
in defect detection, underestimation of defect size, and challenges in capturing magnetic field signal, etc.

Although the high-speed effect influences the performance of the inspection system, it is noteworthy that three distinct fields distribute within the high-speed inspection system, which consist of magnetic leakage field, eddy-current induced field, and residual magnetic field. Each has exhibited prominent advantages in defect detection and characterisation, pattern recognition and material evaluation. In light of this, a high-speed ENDE inspection system has been proposed for evaluation of complex-shaped ferromagnetic rail tracks with surface cracks, inclusions, corrosion, etc, and of coated ferromagnetic pipes.

This case study has shown the advantage of ANSOFT MAXWELL EM in simulating MFL inspection concerning the MFL probe translating over the surface of the magnetic specimen and 3 distributions of fields. It has been found that, in the package, it is straightforward to set up the components with mechanical movement, which benefits the FE simulations for ENDE forward problems involving probes over the samples and the magnetic field signals under the influence of translation of the probes. In COMSOL, the modeling is implementable but more complicated.

4.2 Case study II: FEA for RFEC

The case study discusses the feasibility of COMSOL in modelling the large-dimension component within the FE models for ENDE.

4.2.1 Background

The electromagnetic field for RFEC still complies with the Maxwell’s equations presented in Eqs. (1)-(6). The equation, which is deduced from the Maxwell’s equations and governs the electromagnetic phenomena underlying RFEC, is shown in Eq. (12)
which applies to time-harmonic field since in RFEC the AC excitation current is frequently used.

Eq. (12) can be modified with 2D or 3D notation respectively for 2D or 3D analysis of RFEC problems with the unknown i.e. \( \vec{A} \) rewritten in two or three independent axes. The quantities representing electromagnetic field in RFEC can be calculated after \( \vec{A} \) is computed out via FE approaches.

In order to save computing time, axi-symmetric 2D models of RFEC are usually preferable to 3D models in numerical simulations. \( A_y(r, z) \) has been calculated with fast FEM solvers in previous FEA for RFEC for pipelines without coatings. Nevertheless, the FEA for coated pipelines introduces difficulties in computation, since not only the pipeline but also the coating with its length much larger than the thickness needs to be modelled in the FE simulations. This results in a huge number of triangular elements and makes the subsequent calculation unlikely to succeed. In a bid to overcome the difficulties, quadrangular elements were generated when meshing the problem region of interest, therefore saving computing time, meaning the FE simulations can be conducted using a Pentium processor computer.

The FEA of RFEC of a coated pipeline was implemented using the commercial FEA package, COMSOL, in conjunction with MATLAB. The location of the remote field and RFEC response to surface defect in the coating were investigated respectively.

### 4.2.2 FE simulation setup

As shown in Figure 23, the coated pipeline containing a circumferential surface defect (CSD) was modelled in COMSOL with azimuthal coordinates. The dimension and material specifications, are listed in Table 7, which are set based on a practical coated pipeline [45]:

---

60
Figure 23. The 2D axi-symmetric RFEC model set up in COMSOL

Table 7. Dimension and material of model

<table>
<thead>
<tr>
<th></th>
<th>Inner diameter (mm)</th>
<th>Thickness (mm)</th>
<th>Length (mm)</th>
<th>Conductivity (S/m)</th>
<th>Relative Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pipe</td>
<td>61.4</td>
<td>6</td>
<td>1842</td>
<td>4.032e+6</td>
<td>250</td>
</tr>
<tr>
<td>Coating</td>
<td>125</td>
<td>7.72</td>
<td>1842</td>
<td>4.032e+6</td>
<td>250</td>
</tr>
<tr>
<td>CSD</td>
<td>131</td>
<td>4.72</td>
<td>20</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The outer diameter (OD) of the excitation coil is 44 mm. In consideration of simplification of the model, some assumptions were made:

- Since the dimension of the excitation coil is much smaller than that of the entire model, the excitation coil can be assumed to be a ‘point source’ with its cross-section converging into a point with OD of 44 mm;
- The sinusoidal current through the point source is equivalent to 1 A current through a coil of 5×5 mm² cross-section and 200 turns. The excitation frequency varies from 10 Hz to 100 Hz.

There were two fundamental tasks in FEA. One was to identify the location of the remote field without any CSDs introduced in the specimen, and the other was to investigate the RFEC response to the CSD in coating. For the second task, MATLAB was used as well as COMSOL. The FE simulation was programed in a MATLAB M-file. The signal from
sensors with respect to different positions of excitation coil was obtained in a sequence from the calculation in MATLAB. As shown in Figure 23, the excitation coil starts from the centre of the CSD and travels to the location 38 mm from the CSD centre. 20 simulations were conducted in the MATLAB computation. The simulation procedures are illustrated in the flowchart of Figure 24.

![Flowchart of FEA simulating RFEC response to CSD](image)

**Figure 24. Flowchart of FEA simulating RFEC response to CSD**

4.2.3 *Simulation results and discussions*

4.2.3.1 *Identification of remote field*

Before investigating the RFEC signal response to CSD, the location of remote field at each excitation-frequency case needs to be identified. To fulfil this task, the magnitude and phase of magnetic vector potential (MVP) along the length of pipe, i.e. ‘Base Line’ in Figure 23 were firstly analysed, since the signal from pickup coils is mostly dependent on magnetic vector potential [87]:
\[ v_{emf} = -2 j \omega r A_\phi \]  

(17)

Where, \( v_{emf} \) represents the induced voltage in one turn of the pickup coil with radius of \( r \). \( \omega \) denotes angular frequency of excitation current.

The amplitude of Log of MVP vs. the ratio of distance between the excitation coil and the pickup coil to the OD of pipe at different frequencies cases are presented in Figure 25.

![Figure 25. MVP in Log vs. the ratio of distance between excitation coil and pickup coil to OD of pipe at different frequencies](image-url)

From Figure 25, the general phenomena of RFEC can be found. The MVP as well as signal from pickup coil decrease sharply at first, as the analysis point as well as pickup coil moves away from the excitation coil. Following a ‘pit’ or inflexion point, the rate of decrease in amplitude becomes less. The pits in loci of the signals indicate the transition zones where the field through indirect coupling path couples with the direct magnetic field. The remote field is located adjacent to and just behind the transition zones. Note that the sharp decrease when the ratio of distance between the excitation coil and the pickup coil to the OD of pipe reaches 20 is due to the boundary condition imposed on the edge of the model, giving no indication about remote field.
Pit 1 in Figure 25 implies the transition zone where the field penetrating through the pipe couples with the direct magnetic field. For the coating under investigation, there is another remote field away from the excitation coil with distance of approximately 13 times the pipe OD, which is indicated by Pit 2 as shown in Figure 25. Interestingly, Pit 2 depends much more on excitation frequency. With excitation frequency increased or decreased from the optimal frequency, Pit 2 is masked by the overall amplitude of MVP. These findings indicate there would be an optimal excitation frequency for RFEC inspection of coatings. Using a frequency away from the optimum frequency makes it impossible to evaluate the integrity of the coating from the measured signals.

Figure 26 illustrates the loci of MVP in Log vs. the ratio of distance between excitation coil and pickup coil to OD of pipe for the excitation frequencies of 20 Hz, 40 Hz, 50 Hz, 60 Hz and 70 Hz. From Figure 26, it can be seen that Pit 1 is found at the 20 Hz frequency case while Pit 2 is found at the 40 Hz frequency case. At other frequencies, Pit 2 is rarely located, which makes it unlikely to identify the remote field for coating inspection. Consequently the 40 Hz excitation frequency is used as the optimal frequency, and employed in the following simulations in order to investigate the signal response to CSD.

![Figure 26. MVP in Log against the ratio of distance between excitation coil and pickup coil to OD of pipe at different excitation frequencies](image)

The amplitude and phase of MVP along base line for 40 Hz is shown in Figure 27.
As shown in Figure 27, two transition zones where the fields via indirect coupling paths through pipe and coating couple with the direct magnetic field can be identified by finding the inflexion points at which the abrupt changes can be found in phase of MVP whilst the rate of decline in magnitude of MVP decreases. Both of the plots for the amplitude and phase of MVP indicate the remote fields (after transition zones) for RFEC inspection of the pipe and coating. Accordingly, two analysis points (AP) were chosen for investigation of the signal response to CSD when an excitation coil moves over the CSD, which locate AP1 at 16 times and AP2 at 4 times the pipe OD away from the excitation coil.

4.2.3.2 Signal response to CSD when excitation coil scans over it

Here the CSD was located in the coating instead of the pipe. The variations in magnitude and phase of MVP at AP1 and AP2 when the excitation coil supplied with a 40 Hz sinusoidal current was scanned over the CSD were investigated. The illustration of the scanning and the results are presented in Figure 28. As presented in Figure 28(a), the starting point for the scanning is at the centre of the CSD, at which the position of excitation is set as zero. Note that in this study, the signals when pickup coils at AP1 and AP2 scan over the CSD were not investigated.
Figure 28(b) and Figure 28(c) presents the responses of pickup coils at AP1 and AP2 to the CSD. In general, as shown in Figure 28(b), although the strength of the signal from pickup coil at AP1 is weaker than that at AP2, the variation of amplitude for AP1 is greater than that for AP2 and implies the detection of the CSD. The amplitude for AP2 fluctuates around -2.165e-4 and is unable to indicate the presence of the CSD.

The conclusion drawn from the magnitude plot is reinforced by the phase plot. From Figure 28(c), the phase signal acquired at AP2 fluctuates around 11.15 degrees, which
can be seen insensitive to the CSD. Compared to it, not only is the phase signal at AP1 higher than that at AP2, but also the change of the phase signal at AP1 is greater than that at AP2, which shows the high sensitivity of the pickup coil at AP1 to the integrity of the coating. Furthermore, by deploying two coils at AP1 and AP2, the signals from the coils can be used to distinguish between CSDs in the pipe and CSDs in the coating. A similar conclusion is reported in [122] via experimental study.

4.2.3.3 Comparison of RFEC with Eddy current technique

EC is widely employed in ENDE inspection of metallic specimens. It is likely that EC could be used for inspection of coated pipelines. In order to explore the possible application of EC in the evaluation of a metallic coated pipeline an FE simulation was performed.

The simulation model is identical to that in previous simulations. The sensitivity of RFEC as well as EC to CSF in coating was investigated using Eq. (18) which implies the relative sensitivity (ε) of magnetic field sensors to anomalies in the specimens:

$$\varepsilon = \frac{|A_i - A_0|}{A_0} = \frac{\Delta A}{A_0}$$

(18)

Where, $A_0$ and $A_i$ denote the MVP signals acquired with unflawed and flawed coatings respectively. The excitation frequency for both RFEC and EC varied from 10 Hz to 100 Hz. For RFEC, the measurement of MVP took place at AP1 whilst for EC, MVP was quantified where the excitation coil located. The comparison of $\varepsilon$ between RFEC and EC is shown in Figure 29.
From Figure 29, it can be seen that within this frequency range (10 Hz – 100 Hz), ε for RFEC is much higher than that for EC, which implies that the sensitivity of RFEC to CSF in the coating is much higher than that of EC, even though the excitation frequency of EC is very low. Furthermore, it can also be found in Figure 26 that the optimal frequency with which the RFEC signal has the highest response to CSF is 40 Hz. This finding is consistent with that in Section 4.2.3.1.

The simulation result presents the superiority of RFEC to EC when both of them are applied to the inspection of coated pipelines. The reasoning for this is that in EC the measurement of magnetic field is conducted in the near-excitation zone where the magnetic field comprises the applied field from the excitation coil and the induced field generated by eddy currents in pipeline. In contrast, the field strength measured in remote field attributes to the induced field, which indicates the integrity of eddy current inside the coating. Although the signal strength in the remote field is much lower than that in the near-excitation zone, the measured signal is insusceptible to the applied field. Consequently, it exhibits a high sensitivity to CSF in the coating.

4.2.4 Summary of case study II

The RFEC inspection of metallic coated pipe was simulated using FEM in conjunction with MATLAB. Two remote fields were localised by looking at the characteristics of the
plots of the amplitude and phase of the magnetic vector potential. The first remote field is located behind the excitation coil at a distance of three times the pipe OD and indicates the structural information of the pipe, whilst the second remote field is at a distance of thirteen times the pipe OD away from the excitation and is useful in evaluating the coating due to the weakness of signals. The findings give the implication that two sensors or pick-up coils are to be placed at the two remote fields individually in a bid to acquire the signals, which indicate the structural information of the pipe and coating from the signal amplitude and phase. The long distances between the excitation coil and sensors will be reduced using specific structures in the future [123, 124].

By simulating the same model with different excitation frequencies, it can be found for inspection of the coated pipeline, the selection of frequency plays an important role in detectability of defects in pipes as well as in coatings, although the excitation in low-frequency range is generally well-known for RFEC inspection. For the coated pipe simulated in the FEA, the optimal frequency is 40 Hz and the signals obtained in the first and the second remote field are sensitive to the CSF in the pipe and the CSF in the coating, respectively. This benefits the subsequent signal separation and processing for defect characterisation.

Furthermore, the FE simulation was conducted to compare RFEC with EC when both of them are applied to the inspection of a pipeline with metallic coating. The study shows the superiority of RFEC to EC by investigating the sensitivity of RFEC as well as EC to the CSF in the coating, and indicates that new system structures would be expected for low-frequency EC inspection.

It has been noted in the case study that the configuration of the mesh is flexible and adaptive in COMSOL, which exhibits the advantage of COMSOL over ANSOFT MAXWELL EM. From the FE simulations for RFEC, it is found that the utilisation of a customised mesh can greatly ease the burden of computation on computers so that models with large-size components, which lead to a large number of elements, can be
simulated. Moreover, the integration of COMSOL with MATLAB avails the FEM for ENDE problems regarding probes moving over the specimens under inspection.

4.3 Chapter Summary

Since FEA has shown advantages over the analytical approaches in terms of high flexibility in handling the complex geometrical surfaces of components and the nonlinear magnetic and inhomogeneous materials, this chapter focused on the FE simulations with respect to the ENDE techniques involving MFL and RFEC. Not only the electromagnetic field distribution, but also the magnetic field signals under various inspection conditions are investigated. Through FEA, several outcomes have been achieved: (1) the system design and optimisation are discussed during the post-processing of the simulations; (2) the investigation of magnetic field signals and the features for characterisation of the defects, which are listed in detail as follows:

- The FEA of 3D magnetic field measurement in static MFL inspection of a magnetised sample has been conducted. The study has shown that the signals of $B_y$ provide complementary information for implication of irregular-shaped cracks which can be found in rail tracks. By combining the signals of $B_x$, $B_y$ and $B_z$, the shape of the irregular-shaped crack can be determined. The finding via simulation is verified through experiments of MFL inspection of a section of rail track with a natural crack located near the corner of the rail head. The shape of the crack can be identified by 3D magnetic field measurement using an AMR sensor, which is capable of quantifying the field strength in three independent axes ($x$, $y$, $z$).

- The influence of the velocity of the MFL probe on the acquired MFL signals has been analysed along with the distributions of magnetic flux and eddy current within the system, via FEA. It has been found that the distributions of both magnetic flux and eddy current are distorted, because of which in the obtained MFL signals of $B_x$, the amplitudes of the two peaks are different, whereas the two peaks indicate the direction of the probe translation and its speed. Furthermore, the distributions of three distinct fields have been found in
simulations: (1) leakage field; (2) eddy-current-induced field; (3) residual field.
The finding leads to the optimisation of three-sensor array based dynamic MFL systems implementing the measurement of leakage field, VRM and RFEC. The proposed high-speed MFL system for inspection of rail tracks and pipes integrates the quantification of leakage field with the measurement of eddy-current-induced field and residual field. The sensor arrays instead of signal sensors are to be employed in the system.

- The FE simulations of RFEC inspection of a coated pipe have extended the theoretical study of RFEC, and provided the directions for optimising the RFEC system in terms of the selection of the optimal excitation frequency and the locations where the sensors are deployed. Based on the analysis of the field distribution in magnitude and phase along the length of the pipe, two remote fields are located, which can be used for evaluation and discrimination of the defects in the pipe wall and coating. In addition, the comparison of RFEC with EC in inspection of coated pipes is conducted by analysing the inspection sensitivity to the defects under the identical experimental conditions. RFEC is found advantageous over EC in such a scenario.

Through the three case studies using FEA implemented in COMSOL and ANSOFT MAXWELL EM, it is noted that COMSOL is the preferred package for the FE simulations of SFEC and PEC inspection of stratified conductors, which lies in the facts including: (1) readily defining the components within the FE models; (2) adaptive meshing and flexible adjustment of the mesh condition; (3) implementation of FEA in conjunction with MATLAB to realise the efficient simulations concerning variable parameters; (4) capability of handling models with a moving component even though this is not taken into account in the theoretical study of SFEC and PEC presented in the next chapters.

It is also noted that the FEA is very time-consuming in every case study. The trade-off between computation time and accuracy needs to be made, which is the major drawback of FEA with respect to the efficient ENDE simulation and especially the inverse process.
To achieve fast and accurate modelling, the analytical approach, i.e. ETREE method is proposed. The next chapter will present the details of ETREE modelling particularly for SFEC and PEC simulations. The FE simulations of SFEC and PEC are also illustrated in the next chapter.
CHAPTER 5

ETREE MODELLING OF SFEC AND PEC INSPECTIONS OF MULTILAYERED STRUCTURES

Following the case study of FEM for ENDE inspection involving MFL and RFEC, this chapter focuses on the analytical modelling of SFEC/PEC inspection of stratified conductors using ETREE. The ETREE modelling extends the previous TREE method from simulating the impedance signals from driver coils or pickup coils to predicting the magnetic field signals for solid-state magnetic sensors such as Hall sensors, which have become more widely used in advanced ENDE techniques, such as PEC, due to their advantages over inductive sensors.

5.1 Classic modelling using integral expressions and TREE

5.1.1 Integral expressions

Before dilating the ETREE modelling for SFEC and PEC, an overview of the classic Dodd and Deeds model of integral expressions for EC inspection of multilayered conductors is given. The integral formulation of the expressions have been proven to be capable of predicting the impedance signals from driver coils during the course of EC inspection of multilayered specimens which represent the conductive body (mostly Aluminium alloys) commonly used in in-service mechanical structures such as aircraft wings and metallically-coated pipeline.

Consider the 2D axi-symmetric configuration of a cylindrical coil of rectangular cross-section located above a layered conductor system, as shown in Figure 30.
The magnetic vector potential $\mathbf{A}$ throughout the solution region satisfies [87]:

$$
\begin{cases}
\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} & \text{in the area of vacuum} \\
(\nabla^2 + \gamma^2) \mathbf{A} = 0 & \text{in the non-vacuum region}
\end{cases}
$$

(19)

where, $\nabla^2$ is Laplace operator; $\mu_0$ is permeability of vacuum; $\mathbf{J}$ denotes the current density of the driver coil; $\gamma^2 = j\omega\mu\sigma$; $\mu$ and $\sigma$ are the permeability and the conductivity of the conductor, respectively; $\omega$ is angular frequency, $\omega=2\pi f$, $f$ is excitation frequency.

For the axi-symmetric time-harmonic field in the cylindrical coordinate system, Eq. (19) is rewritten as:

---

Figure 30. A cylindrical coil of rectangular cross-section above a three-layered conductor system
\[
\begin{aligned}
\frac{\partial^2 A(r,z,\omega)}{\partial r^2} + \frac{\partial^2 A(r,z,\omega)}{\partial z^2} + \frac{1}{r} \left[ \frac{\partial A(r,z,\omega)}{\partial r} - \frac{A(r,z,\omega)}{r} \right] &= 0 \\
\frac{\partial^2 A(r,z,\omega)}{\partial r^2} + \frac{\partial^2 A(r,z,\omega)}{\partial z^2} + \frac{1}{r} \left[ \frac{\partial A(r,z,\omega)}{\partial r} - \frac{A(r,z,\omega)}{r} \right] - j\omega\mu\sigma A(r,z,\omega) &= 0
\end{aligned}
\] (20)

Eq. (20) gives the governing equation for electromagnetic phenomena underlying the entire EC system in a cylindrical coordinate system, which can be solved analytically via separation of variables for \(A(r,z,\omega)\) along with the boundary conditions ensuring the continuity of electromagnetic field at the interfaces of different components.

The generalised solution to \(A(r,z,\omega), z>0\) can be expressed as:

\[A(r,z,\omega) = \frac{\mu_0 i_0}{2} \int_0^\infty \frac{J_1(\alpha r)\chi'(\alpha r,\alpha z)}{\alpha^3} \left[ e^{-\alpha z} (e^{-\alpha z} - e^{-\alpha z}) R(a) + F(az_1,az_2,az) \right] da\] (21)

Where, \(J_1(x)\) denotes the first order Bessel function of first order; \(a\) is the separation constant; and the term \(i_0(\omega) = NI/[((r_2 - r_1)(z_2 - z_1))]\) is the source current density of the coil with \(N\) denoting the number of wire turns.

The other terms include the coil term

\[F(az_1,az_2,az) = \begin{cases} 
    e^{\alpha(z_2 - z)} - e^{\alpha(z_1 - z)} & z \geq z_2 \\
    2 - e^{\alpha(z_2 - z)} - e^{\alpha(z_1 - z)} & z_2 \geq z \geq z_1 \\
    e^{\alpha(z_1 - z)} - e^{\alpha(z_2 - z)} & z_1 \geq z \geq 0
\end{cases}\] (22)

and the integral

\[
\chi(x_1,x_2) = \int_{x_1}^{x_2} x J_1(x) dx
\] (23)

This can be calculated by using the identity [125]:

75
\[
\int_{x_1}^{x_2} x J_1(x) \, dx = \frac{\pi}{2} \left\{ x_2 \left[ J_0(x_2)H_1(x_2) - J_1(x_2)H_0(x_2) \right] - x_1 \left[ J_0(x_1)H_1(x_1) - J_1(x_1)H_0(x_1) \right] \right\}
\]  

(24)

Where \( J_0(x) \) is the zero order Bessel function of first kind; \( H_n \) denotes the Struve Function, or alternatively by using the summation relation [125]:

\[
\int_{x_1}^{x_2} x J_1(x) \, dx = \left[ x_1 J_0(x_1) - 2 \sum_{k=0}^{\infty} J_{2k+1}(x_1) \right] - \left[ x_2 J_0(x_2) - 2 \sum_{k=0}^{\infty} J_{2k+1}(x_2) \right]
\]  

(25)

\( R(a) \) is essentially a reflection coefficient that describes the effect of the layered system. Suppose the three-layered system of Figure 30 comprises a top layer having conductivity \( \sigma_1 \), relative magnetic permeability \( \mu_1 \) and thickness \( d_1 \), a middle layer having conductivity \( \sigma_2 \), relative magnetic permeability \( \mu_2 \) and thickness \( d_2-d_1 \) and a bottom layer extending to infinity and having conductivity \( \sigma_3 \) and relative magnetic permeability \( \mu_3 \). In this case [126]:

\[
R(a) = \frac{(a+b_1)e^{-\alpha d_1}U + (a-b_1)V}{(a+b_1)e^{-\alpha d_1}U + (a+b_1)V}
\]  

(26)

Where, for this particular scenario involving the conductor with three layers:

\[
U = (b_1 + b_2)(b_2 - b_3)e^{-2\alpha_1(d_2-d_1)} + (b_1 - b_2)(b_2 + b_3)
\]  

(27)

\[
V = (b_1 - b_2)(b_2 - b_3)e^{-2\alpha_1(d_2-d_1)} + (b_1 + b_2)(b_2 + b_3)
\]  

(28)

Following Eqs. (26)-(28), for a two-layered system, this can be modelled, for example, by setting \( \sigma_2 = \sigma_1 \), the reflection coefficient simplifies to:

\[
R(a) = \frac{(a+b_1)(b_1-b_2)e^{-2\alpha d_1} + (a-b_1)(b_1+b_2)}{(a-b_1)(b_1-b_2)e^{-2\alpha d_1} + (a+b_1)(b_1+b_2)}
\]  

(29)
Whereas for a one-layered system (essentially a conductive half-space modelled by setting $\sigma_3 = \sigma_2 = \sigma_1$) it reduces to:

$$R(a) = \frac{a - b_i}{a + b_i}$$  \hspace{1cm} (30)

In Eqs. (26)-(30), $\alpha_n = \sqrt{\alpha^2 + j\omega\mu_n\sigma_n}$ and $b_n = \alpha_n / \mu_n$, $n=1, 2, 3$.

### 5.1.2 TREE modelling

The numerical calculation of Eq. (21) may present some numerical difficulties, because either appropriate cut-off regions have to be set or automatic integration routines have to be utilised. These difficulties are overcome by a novel approach that replaces the integral expressions with series expansions after truncating the solution region at an appropriate radial distance $r=h$, as shown in Figure 31.

Although the number of series expressions and the radial distance bring about computational errors, the error can be more readily controlled [98]. This approach has been used for the solution of many eddy current NDE problems and is called the TREE method.

By using the TREE modelling, the infinite integral term in Eq. (21) is replaced with series expressions, and thus Eq. (21) is changed to:

$$A(r, z, \omega) = \mu_0 \sum_{i=1}^{\infty} J_1(a_i r) \chi(a_i r, a_i r_z) \left[ e^{-a_i z} (e^{-a_i z_1} - e^{-a_i z_2}) R(a_i) + F(a_1, z_1, a_2, z_2, a, z) \right]$$  \hspace{1cm} (31)

Where, $a_i$ is the discrete eigenvalues derived from the boundary condition at $r=h$, which will be elaborated in the next section. The remaining terms are similar to those in the
integral expressions except substituting $a_i$ for $a$. $\alpha_n = \sqrt{a_i^2 + j\omega \mu_n \sigma_n}$ and $b_n = \alpha_n / \mu_n$, $n=1, 2, 3$. In addition, the index $i$ refers to a particular summation term.

Figure 31. A cylindrical coil of rectangular cross-section above a three-layered conductor system within a truncated region

5.2 ETREE modelling for SFEC and PEC

Instead of predicting the impedance signals using integral expression and TREE modelling, the magnetic field signals are of interest for both SFEC and PEC simulations. Therefore, the solid-state magnetic sensors are introduced within the model. The new model is presented in Figure 32. The stratified conductor consists of arbitrary number of layers with conductivity and permeability of $\sigma_n$ and $\mu_n$, $n=1, 2, 3...L$, respectively. Compared to Figure 31, the solution region is recast so that all the components of the model are in a truncated domain with a radius of $h$. 
Figure 32. A 2D axi-symmetric eddy current model involving a cylindrical coil above a conductor with arbitrary number of layers, a pickup coil and a Hall sensor

The formulation starts with the relationship between the magnetic field $B$ and the magnetic vector potential $A$, which is expressed in Eq. (9). Here, we are concerned with the $z$-component of magnetic field, which is of interest during SFEC and PEC. Therefore, Eq. (30) is rewritten as:

$$
B_z(r,z,\omega) = \frac{1}{r} \frac{\partial [rA(r,z,\omega) \cdot \hat{r}]}{\partial r} \quad \text{for a time-harmonic field}
$$

$$
B_z(r,z,t) = \frac{1}{r} \frac{\partial [rA(r,z,t) \cdot \hat{r}]}{\partial r} \quad \text{for a transient field}
$$

$$
A(r,z,\omega) = \sum_{i=1}^{\infty} J_i(a_i r) \left[ C_0 e^{-\alpha_i z} + D_0 e^{\alpha_i z} \right] \quad \text{for } z \geq 0
$$

$$
A(r,z,\omega) = \sum_{i=1}^{\infty} J_i(\alpha_i r) \left[ C_m e^{-\alpha_m z} + D_m e^{\alpha_m z} \right] \quad \text{for } z < 0
$$

(32)
Where, \( C_{ni} \) and \( D_{ni} \) are the coefficients for the subdomain \( n \), \( n=0 \) (area above the conductor), \( 1, 2...L \); \( \alpha_{ni} = \sqrt{a_i^2 + j\omega\mu_r\mu_n\sigma_n} \), where the eigenvalues of \( a_i \) are the positive roots of the equation depicting the boundary condition at \( r=h \):

\[
J_1(a_i h) = 0 \quad (33)
\]

Or equivalently

\[
J_1(x_i) = 0 \quad ; \quad a_i = x_i / h \quad (34)
\]

It is noted that Eq. (32) applies over the entire modelling region, which make it possible to simulate the total magnetic field distribution, particularly within the areas where in practice the sensors are deployed in order to quantify the field strength.

Before giving the closed-form expressions of magnetic field signals from solid-state sensors, two terms should be specified: point magnetic field (PMF) and volume magnetic field (VMF). PMF denotes the magnetic field at particular points whilst VMF denotes the average magnetic field distributing in a volume governed by the dimension of sensors.

\section*{5.2.1 ETREE modelling for SFEC}

\subsection*{5.2.1.1 Magnetic field at a point \((r, z)\)}

The formulation begins with the derivation of analytical expressions for PMF. The total magnetic field at a point \((r, z)\), \( r \geq 0 \), \( z \geq 0 \) in the air region above the layered conductor system is given by the following equation:

\[
B_z(r, z, \omega) = B_z^{(1)}(r, z, \omega) + B_z^{(2)}(r, z, \omega) \quad (35)
\]
where $B_z^{(1)}(r, z, \omega)$ is the primary magnetic field, namely the field produced by the isolated coil and $B_z^{(2)}(r, z, \omega)$ is the secondary magnetic field depicting the field change caused by the layered conductor.

Following the analysis in [95, 97], whilst taking the recursive property of Bessel function of first kind into account [125]:

$$\frac{d}{dx} [x^\nu J_\nu(x)] = x^\nu J_{\nu-1}(x) \quad \nu \geq 1$$

(36)

The closed-form expression for the $z$-component of the source field depends on the specific region with respect to the axial distance $z$ and is given in general form by:

$$B_z^{(1)}(r, z, \omega) = \mu_0 \delta_0(\omega) \sum_{l=1}^{\infty} J_0(a_i r) \chi(a_i r_1, a_i r_2) F(a_i z_1, a_i z_2, a_i z) a_i^3 \left[ hJ_0(a_i h) \right]^2$$

(37)

The $z$-component of the field change due to the layered system is given by the integral expression:

$$B_z^{(2)}(r, z, \omega) = \mu_0 \delta_0(\omega) \sum_{l=1}^{\infty} J_0(a_i r) \chi(a_i r_1, a_i r_2) e^{-a_i z} \left( e^{-a_i z_1} - e^{-a_i z_2} \right) \frac{V_i}{U_i}$$

(38)

It should be noted that the terms $F(a_i z_1, a_i z_2, a_i z)$ and $\chi(a_i r_1, a_i r_2)$ are given in Eqs. (22) and (23), respectively.

The term $V_i/U_i$ denotes the generalised conductor reflection coefficient which derived from the boundary conditions imposed on the interfaces at $z=0, -d_1, -d_2, \ldots -d_{L-1}$. In an attempt to calculate $V_i/U_i$, two approaches are discussed. In consideration of the boundary condition at those interfaces, a series of linear algebraic equations are obtained, which can be written as:
\[
\begin{align*}
C_0i + D_{0i} &= C_u + D_{li}, \\
k_0 (D_{0i} - C_0i) &= D_{li} - C_i, \\
C_{(n-1)i} + D_{(n-1)i} &= C_{ni} + D_{ni}, \\
k_{n-1} [D_{(n-1)i} e^{-\alpha_{(n-1)i} d_{x}} - C_{(n-1)i} e^{\alpha_{(n-1)i} d_{x}}] &= D_{ni} e^{\alpha_{n_i} d_{x}} - C_{ni} e^{\alpha_{n_i} d_{x}}, \\
k_{n-1} [D_{(L-1)i} e^{-\alpha_{(L-1)i} d_{x}} - C_{(L-1)i} e^{\alpha_{(L-1)i} d_{x}}] &= D_{Li} e^{-\alpha_{L_i} d_{x}}, \\
z &= -d_{n-1} |_{n=2,3,...L-1} \quad (39)
\end{align*}
\]

Where, \( k_0 = \frac{\alpha_{i_1} \mu_i}{\alpha_{i_1} \mu_0} \); \( k_{n-1} = \frac{\alpha_{(n-1)i} \mu_i}{\alpha_{(n-1)i} \mu_{(n-1)}} \);

\[
D_{yi} = \frac{C_{0i}}{V_i} = \frac{\mu_i \omega}{\mu_0} \frac{\chi(a_i r_1, a_i r_2)}{a_i^3 \left[ hJ_0(a_i h) \right]^2}
\]

It is essential to derive the coefficient \( C_{0i} \) such that \( V_i/U_i \) can be acquired. However, it can be found from Eq. (39) that the coefficient matrix of the linear algebraic equations are so sparse that an algorithm based on Singular Vector Decomposition (SVD) [127] is implemented to diagonalise the coefficient matrix and numerically evaluate the solutions to \( C_{0i} \) and subsequently \( V_i/U_i \).

Alternatively, the recursive formulas can be used for computation of the coefficient, which is expressed as [126]:

\[
\begin{align*}
U_n &= \left( \frac{\alpha_{n-1}}{\mu_{n-1}} - \frac{\alpha_n}{\mu_n} \right) e^{-2\alpha_{n} (d_{x} - d_{n})} V_{n+1} + \left( \frac{\alpha_{n-1}}{\mu_{n-1}} + \frac{\alpha_n}{\mu_n} \right) U_{n+1}, \quad n = 2, 3, 4...L-1 \\
V_n &= \left( \frac{\alpha_{n-1}}{\mu_{n-1}} + \frac{\alpha_n}{\mu_n} \right) e^{-2\alpha_{n} (d_{x} - d_{n})} V_{n+1} + \left( \frac{\alpha_{n-1}}{\mu_{n-1}} - \frac{\alpha_n}{\mu_n} \right) U_{n+1} \\
U_L &= \frac{\alpha_{L-1}}{\mu_{L-1}} + \frac{\alpha_L}{\mu_L}; \quad V_L = \frac{\alpha_{L-1}}{\mu_{L-1}} - \frac{\alpha_L}{\mu_L}; \quad \alpha_{n-1} |_{n=1} = \alpha_i; \quad d_0 = 0
\end{align*}
\]

82
During the calculation, the subscript \( n \) iterates from \( L-1 \) to 1.

Eqs. (36)-(38) give the closed-form analytical expressions for magnetic field at a point \((r, z)\), which can be employed to predict the magnetic field signals from the sensors with their dimensions much smaller than the inner radius of the driver coil. In such cases, as illustrated in Figure 32, \( r_0, c_1 \) and \( c_2 \) vanish.

5.2.1.2 Average magnetic field over volume of sensors

It is worth noting that solid-state magnetic field sensors, like the Hall sensor, do not measure the magnetic field at a particular point i.e. PMF, but that within the volume of its sensing element i.e. VMF, under the circumstances that the dimension of the sensing element cannot be negligible with respect to the coil size. Therefore, Eqs. (37) and (38) should be modified to derive the expression of the average magnetic field within the volume of Hall sensing elements.

As can be seen in Figure 32, the length and thickness of the element are not negligible. Firstly, the cube shaped element is converted to a cylinder whilst, its volume and thickness are unchanged. Let the recast cylindrical element have a radius of \( r_0 \) and thickness of \( c=c_2-c_1 \), by taking the volume integration, Eqs. (37) and (38) can be rewritten as:

\[
\phi_1 = \iiint B_z^{(1)} (r, z, \omega) \cdot dv
= 2\pi \mu_0 j_0 (\omega) \sum_{i=1}^{\infty} \frac{\chi(a_i, a_i, r_2)}{a_i^2 \left[J_0^2(a_i h)\right]} \int_0^{r_2} r J_0 (a_i r) F(a_i z_1, a_i z_2, a z) dr dz
\]

\[
\phi_2 = \iiint B_z^{(2)} (r, z, \omega) \cdot dv
= 2\pi \mu_0 j_0 (\omega) \sum_{i=1}^{\infty} \frac{\chi(a_i, a_i, r_2)}{a_i^2 \left[J_0^2(a_i h)\right]} U_1 \int_0^{r_1} \int_{z_1}^{z_2} r J_0 (a_i r) \left[e^{-a_i (z+z_2)} - e^{-a_i (z+z_1)}\right] dr dz
\]
To derive the integral of Bessel function \( rJ_0(a,r) \), the following identity \([125]\) can be used:

\[
\int_0^\infty t^n J_{n-1}(t) dt = x^n J_n(x) \quad n > 0
\] (44)

Consequently, the modified formulations for computing the average magnetic field and its variation due to layered conductors can be written as:

\[
B_{z_1}^{(3)}(\omega) = \frac{\phi_z}{\pi r_0^2 c} = \frac{2\mu_0 i_0(\omega)}{r_0 c} \sum_{i=1}^{N_{\text{a}}} \frac{J_1(a_1 r_0) \chi(a_{i1},a_{i2})}{a_i^2 [hJ_0(a_i h)]^2} \cdot \text{int}_F\]

where,

\[
\text{int}_F = \int_{z_1}^z \mathcal{F}(a_1 z_1,a_1 z_2,a_i z)d(a_i z)
\]

\[
= 2a_1 c + \left[ e^{a_1 (z_2 + z_1 - z_2)} + e^{a_1 z_1} \right] \left( e^{a_1 z_2} - e^{-a_1 z_1} \right)
\]

\[
= \left( e^{-a_1 z_1} - e^{-a_1 z_2} \right) \left( e^{a_1 z_2} - e^{a_1 z_1} \right)
\]

\[
z_2 > a_1 z_1 \geq a_1 z_2
\]

\[
z_1 > a_1 z_1 \geq 0
\]

\[
B_{z_1}^{(2)}(\omega) = \frac{\phi_z}{\pi r_0^2 c}
\]

\[
= \frac{2\mu_0 i_0(\omega)}{r_0 c} \sum_{i=1}^{N_{\text{a}}} \frac{J_1(a_1 r_0) \chi(a_{i1},a_{i2}) V_i}{a_i^2 [hJ_0(a_i h)]^2} \cdot \left[ e^{-a_{i1}(z_2 + z_1)} - e^{-a_{i1}(z_1 + z_2)} - e^{-a_{i1}(z_1 + z_3)} - e^{-a_{i1}(z_1 + z_3)} \right]
\]

(47)

Eq. (47) can be written in a more compact form as:

\[
B_{z_1}^{(3)}(\omega) = \frac{\phi_z}{\pi r_0^2 c} = \frac{2\mu_0 i_0(\omega)}{r_0 c} \sum_{i=1}^{N_{\text{a}}} \frac{J_1(a_1 r_0) \chi(a_{i1},a_{i2}) V_i}{a_i^2 [hJ_0(a_i h)]^2} \cdot \frac{V_i}{U_i} \left( e^{-a_{i1} z_2} - e^{-a_{i1} z_1} \right) \left( e^{-a_{i1} z_2} - e^{-a_{i1} z_1} \right)
\]

(48)
Thus, the averaged net magnetic field within the volume of the Hall sensor, $B_{zv}$ is derived from superimposing the primary magnetic field $B^{(1)}_{zv}$ and the secondary magnetic field $B^{(2)}_{zv}$, which can be expressed as:

$$B_{zv}(\omega) = B^{(1)}_{zv}(\omega) + B^{(2)}_{zv}(\omega)$$  \hspace{1cm} (49)$$

Eqs. (45)-(49) give the expression of ETREE for the magnetic field signals from solid-state magnetic field sensors such as Hall sensors, GMR, etc for SFEC, which can be readily computed with moderate effort using mathematical packages such as Mathematica [128] or MATLAB. Mathematica also has intrinsic routines for calculating Bessel function roots and the integral in Eq. (33).

In Eq. (48) only the $V_I/U_I$ depends on the excitation frequency. When the excitation frequency sweeps within a certain range, $V_I/U_I$ has to be computed for every frequency and other terms only once. This results in a substantial saving of computation time. Such a saving is significant, because what is required in various inversion schemes is a fast forward solution in terms of the frequency. Thus, the numerical computation of the magnetic field as a function of frequency, for a sweep-frequency scan, takes no more than 1s in a typical computer with an Intel® Core™2 Duo processor.

### 5.2.2 ETREE modelling for PEC

Since a transient signal, such as a pulsed excitation current in PEC, can be theoretically represented by superimposing a series of sinusoidal harmonics in the frequency domain, the transient magnetic field signal for a multilayered sample can be derived from a sum of time harmonic responses to the sample in frequency domain by using either inverse Fourier Transform (IFT) [129, 130] or inverse Laplace Transform (ILT) [131]. Here, we focus on the IFT, which is employed in computation in light of its distinct advantage over ILT that it is readily approximated efficiently and numerically by using inverse
Fast Fourier Transform (IFFT) [127] that has already been implemented in several scientific computation packages such as MATLAB and Mathematica.

Following the derivation of Eqs. (45) and (48) which give the expressions of magnetic field against various frequencies, the magnetic field as a function of time can be recovered and written mathematically in a Fourier manner as:

\[
B_{2v}(t) = B_{2v}^{(1)}(t) + B_{2v}^{(2)}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ B_{2v}^{(1)}(\omega) + B_{2v}^{(2)}(\omega) \right] e^{j\omega t} d\omega
\]

\[
B_{2v}^{(1)}(t) = \frac{2\mu_0 I_0(t)}{r_0 c} \sum_{j=1}^{\infty} \frac{J_1(a_j r_0)}{a_j^2 \left[ h J_0(a_j h) \right]^2} \cdot \text{int}_F
\]

\[
B_{2v}^{(2)}(t) = \frac{2\mu_0}{\pi r_0 c} \sum_{j=1}^{\infty} \frac{J_1(a_j r_0) \chi(a_j r_1, a_j r_2) \left( e^{-a_j r_2} - e^{-a_j r_1} \right) \left( e^{-a_j^2 z^2} - e^{-a_j^2 z_0^2} \right)}{a_j^2 \left[ h J_0(a_j h) \right]^2} \int_{-\infty}^{\infty} i_0(\omega) \frac{V_i}{U_i} e^{j\omega t} d\omega
\]

\[
i_0(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} i_0(\omega) e^{j\omega t} d\omega
\]

where, \( i_0(t) \) denotes the transient excitation current such as a pulsed current; \( B_{2v}(t) \), is the net transient magnetic field which is the superposition of the primary field \( B_{2v}^{(1)}(t) \) and the secondary field \( B_{2v}^{(2)}(t) \).

Eq. (50) gives the generalised analytical expression of the transient magnetic field signal picked up by a solid-state magnetic field sensor. It is apparent that, if the sensor dimension is negligible, Eq. (50) can be simplified into the expression for PMF:
\[
B_z(t) = B_z^{(1)}(t) + B_z^{(2)}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \left[ B_z^{(1)}(\omega) + B_z^{(2)}(\omega) \right] e^{j\omega t} d\omega
\]

\[
B_z^{(1)}(t) = \mu_0 i_0(t) \sum_{n=1}^{\infty} \frac{J_0(a_r) \chi(a_z) F(a_z \xi, a_z \eta)}{a_z^3 [hJ_0(a_z)^2]^{3/2}} i_0(\omega) \int_{-\infty}^{\infty} i_0(\omega) \frac{V_I}{U_i} e^{j\omega t} d\omega
\]

\[
i_0(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} i_0(\omega) e^{j\omega t} d\omega
\]

In order to mitigate the computational effort in the evaluation of the unbound integral, IFFT has been employed. As a result, the transient magnetic field signal acquired by a solid-state magnetic field sensor is expressed in the time domain as:

\[
B_z(t) = \frac{1}{n} \sum_{\omega_0=1}^{n} \left[ B_z^{(1)}(\omega) + B_z^{(2)}(\omega) \right] W_n^{-(j-1)(\omega-1)}
\]

Where, \( W_n = e^{-(j2\pi)/n} \) is the \( n \)th root of unity.

### 5.2.3 Consideration of ETREE modelling for PEC

The analytical modelling of PEC using ETREE is straightforward, because Eq. (50) still applies to the modelling of the pulsed magnetic field, which is a particular case of a transient field.

However, problems arise when considering the excitation current that is input into the ETREE model for PEC. In previous modelling, the excitation current is firstly predicted based on the electrical characteristics of the driver coil such as inductance, capacitance and resistance, before it is introduced into the simulation models, particularly the analytical models. The predicted excitation current can be expressed as [131]:

\[
I(t) = \ldots
\]
where, \( U \) and \( R \) denote the voltage across the driver coil and the coil resistance. \( u(t) \) is a sign function of time. Note that the time constant \( \tau_0 \) within Eq. (53) is highly dependent of the driver coil’s electrical characteristics, which are acquired before conducting the modelling. Nevertheless, to determine the electrical characteristics needs the precise measurement.

During the course of the theoretical study, the excitation current put into the ETREE model was obtained during practical experiments in order to avoid the discrepancy between the predicted current and measured current, and consequently the discrepancy between the simulated PEC signals and measurement results. The modelling procedures are shown in Figure 33.

Figure 33. The procedures of ETREE modelling for PEC (\( \omega_k \) denotes the frequency harmonics within pulsed excitation current; \( k \) denotes \( k^{th} \) harmonic)
5.3 Corroboration of ETREE model

It is imperative to verify the validity of the ETREE modelling for SFEC and PEC inspections on stratified conductors. FE simulations and experiments have been carried out to provide the comparative results with those from ETREE modelling. Two types of layered structures were employed in the comparison work, which are shown in Figure 34.

![Figure 34. SFEC/PEC inspection of two stratified structures: (a) Two-layer structure (Structure 1); (b) Three-layer structure (Structure 2)](image)

5.3.1 FE simulations for SFEC and PEC inspections on multilayered structures

In addition to analytical modelling, eddy current phenomena were also investigated via numerical simulations including FEM. The FE simulations were carried out to compare the accuracy and efficiency of ETREE. Two-dimensional axi-symmetric models were built up for different layered structures. The FEM program was programmed in MATLAB in conjunction with COMSOL.

Because FEM is a mesh-dependent method, a dense mesh was chosen, especially within the cross-sections of the coil, Hall sensing element and layered samples to obtain the converged results. The 2D axi-symmetric FE model for Structure 2 (upper layer: Aluminium; medium layer: Brass; bottom layer: Air) is shown in Figure 35.
Figure 35. Close view of a 2D axi-symmetric FE model for Structure 2 (areas in red represent air)

The models were applied to two time-dependent solvers, time-harmonic solver and time-stepping solver in order to solve the FEM equations and predict the magnetic field signals picked up by a Hall sensor for time-harmonic field and transient field, respectively. The excitation current input into the model was acquired from practical measurements, including: current in terms of maximum amplitude and frequency for SFEC simulations and current amplitude as function of time for PEC simulations.

5.3.2 Experimental study

The z-component of the magnetic field above a multilayered conductive structure (MCS) was measured using a Hall sensor with its sensitivity axis perpendicular to the surface of the multilayered structure. The measurements are compared with both analytical and numerical results.

The schematic of the experimental setup is shown in Figure 36 with the driver coil presented in Figure 37. A waveform generator and a power amplifier were employed to power the driver coil with a sinusoidal current and transient current (rectangular waveform). Note that the coil power source was a constant voltage source, which kept the output peak-to-peak voltage at a preset voltage. The amplitude of the current through the driver coil was monitored and controlled by adjusting the voltage output.
from the source. In order to avoid the coil thermal drift during excitation, for the study of SFEC the current amplitude was set at 91.5 mA (maximum value at lowest frequency). 27 discrete frequency values were selected within the range from 20 Hz to 10 kHz. Regarding the study of PEC the maximum amplitude and repetition frequency of the current (of rectangular waveform) were 50 mA and 140 Hz, respectively.

Figure 36. Schematic experimental setup

Figure 37. Driver coil with a Hall sensor

A Hall sensor (Honeywell, SS495A, Measurement range from -0.067 T to 0.067 T, Sensitivity of 3.125 mV/Gauss) was used to evaluate the z-component of the net magnetic field and placed at the centre of the driver coil. The calibration of the Hall
sensor was implemented by comparing the output from the sensor when the probe was put in the air with the theoretical value from the classic modelling using integral expressions. A circuit power source was adopted to power the Hall sensor as well as the signal conditioning circuit where an INA111 high-speed FET-input instrumentation amplifier was chosen. The pre-processed signals were then obtained by a data acquisition card, controlled by a signal-acquisition-and-processing program implemented in LabVIEW.

The dimensions and properties of the layers and the driver coil employed in the simulation and experimental studies are shown in Table 8 and Table 9, respectively.

### Table 8. Dimension and properties of each layers

<table>
<thead>
<tr>
<th>Layers Material</th>
<th>Conductivity* (S/m)</th>
<th>Relative Permeability</th>
<th>Length (mm)</th>
<th>Width (mm)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>3.4e7</td>
<td>1</td>
<td>100</td>
<td>99.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Brass</td>
<td>1.4e7</td>
<td>1</td>
<td>400</td>
<td>600</td>
<td>9.5</td>
</tr>
</tbody>
</table>

*The conductivities of all non-ferromagnetic samples were measured with a GE AutoSigma 3000.

### Table 9. Coil parameters

<table>
<thead>
<tr>
<th>Outer diameter (OD) 2r₂ / mm</th>
<th>Inner diameter (ID) 2r₁ / mm</th>
<th>Length z₂-z₁ / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.64±0.01</td>
<td>22.57±0.01</td>
<td>6.62±0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of turns N</th>
<th>DC Resistance R / ohm</th>
<th>DC Inductance L / mH</th>
<th>Design Lift-off z₁ / mm</th>
<th>Copper Wire Diameter Φ / mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>804±1</td>
<td>134.7</td>
<td>19.4</td>
<td>0.64±0.01</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The geometrical parameters of the coil were measured with a vernier caliber. The copper wire diameter was given by manufacturer.

The lift-off of the excitation coil is determined by the actual thickness of the coil former. The DC inductance and resistance of the coil were measured using an LCR Bridge (measurement error less than 0.1%) with a frequency of 100 Hz. The measured coil inductance shows a discrepancy of 1% from the theoretical value of 19.6 mH, which was derived from the classic modelling using integral expressions. The resonance frequency of the excitation coil is 250.5 kHz.
After destructive investigation to remove the chip silicon package, the dimension of the sensing element of the Hall sensor was found to be $0.91 \times 0.46 \text{ mm}^2$. Because its centre is located at the centre of the driver coil, 0.5 mm above the surface of the specimen ($c_f=0.5 \text{ mm}$), the second and third expressions of Eq. (46) are employed in ETREE modelling.

### 5.3.3 Comparison for SFEC

#### 5.3.3.1 Computation accuracy

The results obtained from ETREE, FEM and experiment are compared in terms of (1) magnetic field as function of frequency and (2) magnetic field per unit excitation current as function of frequency, which is used to evaluate the efficiency of excitation current for field response to the samples. The comparison results are shown individually in Figure 38 and Figure 39.

![Figure 38. Magnetic field vs. excitation frequency in logarithmic scale for (a) Structure 1 and (b) Structure 2 (markers represent actual data points)](image-url)
Figure 39. Magnetic field per unit excitation current vs. excitation frequency in logarithmic scale for (a) Structure 1 and (b) Structure 2 (markers represent actual data points)

In general, the predicted magnetic field signals over the excitation frequency range via ETREE and FEM show good agreement with the experimental results. The difference between experimental test, analytical model (ETREE) and numerical model (FEM) is within 1%, which is at almost the same level as conventional modelling for impedance signal [92, 100]. Unlike coil based impedance signals, the phase variation of the magnetic field was not investigated and the results are only presented in terms of the magnitude of the magnetic field.

The results of theoretical study, in the frequency range of 1 kHz to 10 kHz, agree well with the experimental results. Nevertheless, the difference between theory and experiment is higher for Structure 2 than Structure 1. This small difference may be because there is a very small gap (approximately 0.1 mm) between the Aluminium plate and the Brass plate, while in the theoretical model no gap is assumed. In further work, if a thin air layer were modelled between the two conductors, the discrepancy would be cancelled.

5.3.3.2 Computation time

The computation time is one of the most important factors used in evaluating the efficiency of simulations. Both the ETREE modelling and the FE simulation are
conducted on a computer with an Intel® Core™2 Duo 2.13 GHz CPU, 1GB RAM. The number of computed frequencies was 27 from 20 Hz to 10 KHz. The time taken by the ETREE and FE simulations is shown in Table 10.

<table>
<thead>
<tr>
<th></th>
<th>2-layer case</th>
<th>3-layer case</th>
</tr>
</thead>
<tbody>
<tr>
<td>ETREE</td>
<td>2.85s</td>
<td>4.67s</td>
</tr>
<tr>
<td>FEM</td>
<td>262.63s</td>
<td>411.06s</td>
</tr>
</tbody>
</table>

The analytical approach provides the compact closed-form formulations of magnetic field signals, which is entirely mesh-independent. In contrast, a highly dense mesh was chosen to obtain converged results when FE simulations are conducted, which made FEM a lot more time consuming than ETREE.

Table 10 shows that ETREE is much more efficient than FEM, even though the FE simulations are implemented in MATLAB to save computation time. It is apparent that ETREE exhibits superiority over FEM, as ETREE provides a compact closed-form solution to forward time-harmonic problems. Moreover, most parameters in the formulations need to be computed only once over the frequency domain and this makes the computation of eigenvalues and coefficients of series expansions very fast.

5.3.4 Comparison for PEC

5.3.4.1 Computation accuracy

The temporal PEC signals with respect to 2-layer and 3-layer structures are compared between experiment and theory viz. ETREE and TSFEM, which are plotted in Figure 40. The PEC differential signal for each structure case were derived from the subtraction of signals with and without conductors, and presented in Figure 41.
As can be seen in Figure 40 and Figure 41, compared to TSFEM, the results from ETREE modelling for PEC have much better agreement with those from experiments in terms of PEC signal with respect to each stratified conductor and PEC differential signals. The reason lies in the fact that ETREE modelling is free from the meshing problems that crucially affect the accuracy of solutions from TSFEM. In order to compare quantitatively the accuracies of ETREE and TSFEM, the Normalised Root Mean Squared Deviations (NRMSD) between theory and experiment were calculated over all time steps. The computation of NRMSD is shown in Eq. (53) [132]:
where, index $i$ denotes the time instant; $x_{p,i}$ and $x_{m,i}$ are the predicted and measured values at the time instant $i$, respectively; $x_{\text{max}}$ and $x_{\text{min}}$ are the maximum and minimum values of the measured signals over the period, respectively.

Although the noise in measured current brought about the fluctuation of predicted signals, the NRMSD between ETREE and experiment is less than 3%, as shown in Table 11. In contrast, Table 12 shows the higher NRMSD between TSFEM and experiment, indicating a greater error when compared to ETREE.

**Table 11. NRMSD between ETREE and experiment**

<table>
<thead>
<tr>
<th></th>
<th>Air</th>
<th>Structure 1</th>
<th>Structure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEC signal</td>
<td>1.4%</td>
<td>0.9%</td>
<td>2.0%</td>
</tr>
<tr>
<td>Differential signal</td>
<td>N/A</td>
<td>2.0%</td>
<td>2.8%</td>
</tr>
</tbody>
</table>

**Table 12. NRMSD between TSFEM and experiment**

<table>
<thead>
<tr>
<th></th>
<th>Air</th>
<th>Structure 1</th>
<th>Structure 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PEC signal</td>
<td>1.4%</td>
<td>2.0%</td>
<td>4.1%</td>
</tr>
<tr>
<td>Differential signal</td>
<td>N/A</td>
<td>7.3%</td>
<td>8.7%</td>
</tr>
</tbody>
</table>

For PEC inspection, peak value (the maximum value of the differential signals) and peak time (the time instant corresponding to peak value) in differential signals play an important role in the characterisation of each layer, and are taken as essential features for the inverse process. Further investigation of these two features of the differential signal presents better consistency between ETREE and experiment, and gives the conclusion that ETREE is feasible and useful for the simulation of PEC in the time domain with a high accuracy.

Nevertheless, from Figure 40 and Figure 41, it can be seen that there are discrepancies in the plots between the theoretical and experimental results, especially in Figure 41,
where the greatest difference occurs at the early stages of excitation when the high-frequency component is dominant. The reasons for this are as follows:

- Although PEC signals can be recovered by FFT, the pitfall of FFT, namely Gibbs phenomena \[133\] which causes oscillations in the proximity of discontinuity points in the PEC waveform contributes to the discrepancy;

- At the early stage of excitation, high-frequency harmonics are in majority of the spectrum. In the simulation, all of the frequency components including those high-frequency harmonics are taken into account. However, the Hall sensor has a frequency-response band covering DC to 10 kHz. The frequency harmonics over 10 kHz actually give little contribution to the measured PEC signals. Therefore, the discrepancies are attributed partly to the fact that the frequency-response characteristics of the Hall sensor is not dealt with in the theoretical models;

- The 1% difference in DC inductance of the excitation coil between theory and experiment is also a cause of discrepancy;

- Particularly for Structure two, there is a very small gap (approximately 0.1 mm) between the Aluminium plate and Brass plate, while in the theoretical model no gap is assumed. The difference causes the discrepancies at the time range up to 0.1 ms.

5.3.4.2 Computation time

The computing time spent on ETREE was evaluated and compared with TSFEM, which was conducted in COMSOL, shown in Table 13.

<table>
<thead>
<tr>
<th>Modelling Cases</th>
<th>ETREE</th>
<th>TSFEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computing time (approx.)</td>
<td>0.53s</td>
<td>378.08s</td>
</tr>
<tr>
<td>Other layered cases</td>
<td>1.05s</td>
<td>791.08s</td>
</tr>
</tbody>
</table>

\[\text{Number of elements (averaged): 42,515}\]
From the comparison results, it can be seen that the computation speed of ETREE is faster than that of TSFEM, which is consistent with the comparison results for SFEC. Although ETREE modelling for PEC is based on the time-harmonic simulation in contrast to TSFEM, which implements the simulation in the time domain, it can still be found superior to TSFEM in terms of computation accuracy and time. This is attributed to the advantages of ETREE that it is mesh-independent and most coefficients in expressions are frequency-independent.

5.3.5 Concluding remarks

From the comparison study for SFEC and PEC between theory and experiment, new models for magnetic field measurement, rather than traditional detection coils are investigated and analysed via simulations of magnetic field responses to different multilayered structures. Based on the results, ETREE has advantages in the solution for magnetic-sensor-based EC forward problems, which demands the modelling of the Hall sensor, which has a finite volume. The conclusions drawn are as follows:

- A good agreement can be found among ETREE, FEM and experimental tests, which fulfils the requirements from more and more EC inspection systems using magnetic sensing systems for NDT&E.
- ETREE is mesh-independent and offers closed-form formulations, so magnetic field signals can be derived effectively and accurately. The ETREE modelling can be beneficial for (1) the design and development of eddy current NDT&E systems; (2) the acquisition of reference/calibration magnetic field signals via ETREE modelling and without practical reference/calibration samples (3) in the inverse modelling process, in characterising of each layer, including conductivity and thickness, that is, the layered structure’s conductivity profile.

5.4 Chapter summary

This chapter elaborates the ETREE modelling of SFEC and PEC, which involves (1) the formulation of the closed-formed expressions of magnetic field signals from solid-state
magnetic field sensors in time domain and frequency domain; (2) the verification of ETREE modelling by comparing the simulation results via ETREE with experiment and FEM.

From the investigation, it is note worthy that ETREE modelling is superior to FEM in terms of (1) up to 5% higher computation accuracy and (2) up to 99% faster simulation time. This is due to the fact that ETREE modelling is entirely mesh-independent, and gives the compact expressions of magnetic field signals for SFEC and PEC with most coefficients frequency-independent. Compared with the other analytical methods i.e. integral expression method and TREE method, ETREE method eases of computation since the infinite integrals in previous analytical expressions are replaced with a series of eigenfunction expansions, and is more flexible in adjusting the computational error by choosing the adequate number of expansions and eigenvalues. It is also note worthy that the ETREE modelling takes the dimension of a solid-state magnetic field sensor into account, which makes the method more accurate in predicting the magnetic field signals and feasible in simulating SFEC/PEC inspection along with solid-state magnetic field sensors.

The work opens up an inverse model based on ETREE, which will be presented in the next chapter.
CHAPTER 6

INVERSE PROCESS FOR LIFTOFF ESTIMATION AND
CONDUCTIVITY EVALUATION USING LOI

In the previous chapter, ETREE modelling for SFEC and PEC is given and the closed-form expressions for magnetic field signals from solid-state magnetic field sensors are derived. In this chapter, the analytical equation for LOI is derived based on ETREE in order to give an insight into the characteristics of LOI in SFEC and PEC. ETREE is also made capable of predicting the magnetic field signals from sensors at arbitrary locations above the surface of samples. It has been found that with multiple lift-offs introduced, LOI shows more range characteristics than a point, and it is dependent on the parameters of inspection systems. Subsequently, inverse schemes based on LOI for estimation of the conductivity of a conductive half-space and the lift-off of a probe is proposed in conjunction with the database established via ETREE.

6.1 Analytical formulation of LOI with sensors at the centre of driver coil

This section gives the analytical expression of LOI when the magnetic field sensor is placed at the centre of the driver coil.

6.1.1 The net magnetic field

Following the derivation of Eqs. (45)-(48), Eq. (49) is rewritten by integrating the expressions of the primary magnetic field and the secondary magnetic field:
Where, \( \text{int}_F_i \) is expressed in Eq. (46); \( V_j/U_j \) is computed using Eq. (41); and \( W_i \) is written as follows:

\[
W_i = \begin{cases} 
  e^{-a_i(z_2 + z_1)} & c_2 > c_1 \geq z_2 \\
  e^{a_i z_2} - e^{a_i z_1} & z_2 \geq c_2 > c_1 \geq z_1 \\
  2a_i c e^{a_i(z_1 + z_2)} + e^{a_i(z_1 + z_2)} \left( e^{a_i(c_1 + z_2)} + e^{a_i(c_1 + z_2)} \right) e^{-a_i(c_2 + z_1)} & z_1 \geq c_2 > c_1 \geq 0
\end{cases}
\]  

(56)

Subsequently, the transient net magnetic field, which is expressed in Eq. (50), can be modified as:

\[
B_{z_2}(t) = \sum_{i=1}^{\infty} M_i \cdot \text{int}_F_i \left[ i_0(t) + \frac{W_i}{\pi} \int_{-\infty}^{\infty} i_0(\omega) \frac{V_j}{U_1} e^{i\omega t} d\omega \right] 
\]

(57)

Note that in Eqs. (55) and (57), \( V_j/U_j \) is in function of frequency, and introduces complex values to the solutions of magnetic field signals. The complex values contribute the emergence of LOI within the signals of both SFEC and PEC when the lift-off varies.

### 6.1.2 Formulation of LOI for SFEC

The model used in the investigation is shown in Figure 32 with an EC probe placed over a stratified conductor with arbitrary number of layers. The radial position of the
magnetic field sensor is at the centre of the driver coil, and its vertical position varies within the range $c_1 \geq 0$, $c_2 > c_1 > 0$.

It is worth noting that in Eq. (56) only the variables $c_1$, $c_2$, $z_1$ and $z_2$ are dependent on lift-off. Let $lo$ denote lift-off variation ($lo \neq 0$). Substitute $c_1' = c_1 + lo$, $c_2' = c_2 + lo$, $z_1' = z_1 + lo$ and $z_2' = z_2 + lo$, for $c_1$, $c_2$, $z_1$ and $z_2$ in Eq. (56), thus we have:

$$W_i = \begin{cases} e^{-a_i(z_2 + z_1 + 2lo)} & c_2 > c_1 \geq z_2 \\ \frac{e^{a_i z_1} - e^{a_i z_2}}{2a_i e^{a_i z_1 + u_0} + e^{a_i (z_1 + z_2 + 2lo)} + e^{a_i (z_1 + z_2 + 2lo)}} & z_2 \geq c_2 > c_1 \geq z_1 \\ e^{-a_i(z_2 + z_1 + 2lo)} & z_1 \geq c_2 > c_1 \geq 0 \end{cases}$$  (58)

Thus, the magnetic field signal after introducing lift-off variation becomes:

$$B_z(\omega) = i_0(\omega) \sum_{i=1}^{\infty} M_i \cdot \text{int}_{-F_i} \left[ 1 + \frac{V_1}{U_1} \cdot W_i \right]$$  (59)

LOI feature occurs in general, when the condition $B_z(\omega) = B'_z(\omega)$ applies, which indicates that to derive the LOI the root of the condition needs to be found.

Following [133], the time instant when LOI occurs ($T_{loi}$) can be expressed as:

$$T_{loi} = \frac{1}{\omega} \tan^{-1} \left\{ \frac{\Im[B_z(\omega)] - \Im[B'_z(\omega)]}{\Re[B_z(\omega)] - \Re[B'_z(\omega)]} \right\}$$  (60)

Where, $\Re$ and $\Im$ denote the real and imaginary parts respectively. Therefore, the derivation of real and imaginary parts of Eqs. (55) and (59) is the key to the solution of $T_{loi}$. The term $V_1/U_1$ essentially gives complex values within the solution, thus the separation of real and imaginary parts begins with the modification of $V_1/U_1$. 

103
Here, firstly we simplify the model and change the stratified conductor into a conductive half-space with permeability \( \mu, \mu = \mu_0 \mu_r \) and conductivity \( \sigma \). The simplified model is presented in Figure 42 with the dimension of a solid-state sensor taken into account. Subsequently the term \( V_i/U_i \) can be simplified to:

\[
\frac{V_i}{U_i} = \frac{a_i \mu_i - \sqrt{a_i^2 + j \omega \mu_0 \mu_i \sigma}}{a_i \mu_i + \sqrt{a_i^2 + j \omega \mu_0 \mu_i \sigma}}
\]

\[
= \frac{a_i^2 (\mu_i^2 + 1) + j \omega \mu_i \sigma - 2a_i \mu_i \sqrt{a_i^2 + j \omega \mu_i \sigma}}{a_i^2 (\mu_i^2 - 1) - j \omega \mu_i \sigma}
\]

(61)

---

**Figure 42.** A 2D axi-symmetric eddy current model involving a cylindrical coil, a conductive half-space and a Hall sensor

Eq. (61) includes the square root of a complex term. By using the formula:

\[
\sqrt{x + jy} = \frac{r + x}{\sqrt{2(r + x)}} + \frac{jy}{\sqrt{2(r + x)}}
\]

\[
r = |x + jy| = \sqrt{x^2 + y^2}
\]

(62)
Eq. (61) is written as:

\[
\frac{V_i}{U_i} = \Re(V_i/U_i) + j\Im(V_i/U_i)
\]

\[
\Re(V_i/U_i) = \frac{a_i^2 (\mu_i^2 - 1) \left[ a_i^2 (\mu_i^2 + 1) - G_i \right]}{a_i^2 (\mu_i^2 - 1) + (\omega \mu_i \mu_0 \sigma)^2} \left[ 1 - H_i \right]
\]

\[
\Im(V_i/U_i) = \omega \mu_i \mu_0 \sigma \frac{a_i^2 (\mu_i^2 - 1) \left[ 1 - H_i \right] + a_i^2 (\mu_i^2 + 1) - G_i}{a_i^2 (\mu_i^2 - 1) + (\omega \mu_i \mu_0 \sigma)^2}
\]

\[
G_i = 2a_i \mu_i \sqrt{\frac{R_i + a_i^2}{2}}
\]

\[
H_i = \frac{2a_i \mu_i}{\sqrt{2(R_i + a_i^2)}}
\]

\[
R_i = \sqrt{a_i^2 + (\omega \mu_i \mu_0 \sigma)^2}
\]

If \( \mu = 1 \), Eq. (63) is simplified and rewritten as:

\[
\frac{V_i}{U_i} = \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) + \frac{2 j a_i}{\omega \mu_0 \sigma} \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right)
\]

Thus, the expression of magnetic field can be written in complex manner as:

\[
\begin{align*}
B_{zv}(\omega) &= \Re[B_{zv}(\omega)] + j\Im[B_{zv}(\omega)] \\
\Re[B_{zv}(\omega)] &= i_0(\omega) \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \left[ 1 + \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) W_i \right] \\
\Im[B_{zv}(\omega)] &= \frac{2i_0(\omega)}{\omega \mu_0 \sigma} \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \left[ 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right] W_i
\end{align*}
\]

The expression of magnetic field after introducing the lift-off variation is readily obtained by substituting \( W_i \) for \( W_i \)
Substituting Eq. (65) into Eq. (60), the analytical expression of the time when LOI takes place for a time-harmonic field is expressed as:

$$T_{\text{lo}} = \frac{1}{\omega} \tan^{-1} \left( \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_i F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) (W_i - W_i')}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_i F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) (W_i - W_i')} \right)$$

(66)

Suppose the Hall sensor is placed in the region $z_1 \geq c_2 > c_1 \geq 0$. Eq. (66) is rewritten as:

$$T_{\text{lo}} \bigg|_{z_1 \geq c_2 > c_1 \geq 0} = \frac{1}{\omega} \tan^{-1} \left( \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_i F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) \left( 1 - e^{-2a_i \omega_0} \right) e^{-a_i (c_1 + c_2)}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_i F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) \left( 1 - e^{-2a_i \omega_0} \right) e^{-a_i (c_1 + c_2)}} \right)$$

(67)

In a similar way, when the position of the Hall sensor is changed to $c_2 > c_1 \geq z_2$; $z_2 \geq c_2 > c_1 \geq z_1$, Eq. (66) is then written as:

$$T_{\text{lo}} \bigg|_{z_2 > c_2 > z_1} = \frac{1}{\omega} \tan^{-1} \left( \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_i F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) \left( 1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_i F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) \left( 1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)}} \right)$$

(68)
Eqs. (66)-(69) give the analytical expression for the time instant when the LOI occurs after the lift-off variation is introduced during the course of SFEC inspection, on a non-magnetic conductive half-space.

It should be noted that the expression of LOI for a stratified conductor could be derived from the separation of real and imaginary parts of $V_j/U_j$. However, since the formulation of $V_j/U_j$ for a layered structure with arbitrary number of layers is complicated, and the separation process is intricate, therefore, only the formulation for the conductive half-space is focused on.

### 6.1.3 Formulation of LOI for PEC

The closed-form expression of the net transient magnetic field of PEC is given in Eq. (57). Suppose that the lift-off variation is introduced, substitute $c_1' = c_1 + lo$, $c_2' = c_2 + lo$, $z_1' = z_1 + lo$ and $z_2' = z_2 + lo$, for $c_1$, $c_2$, $z_1$ and $z_2$ in Eq. (57), we have:

$$B_3(t) = \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \left[ i_i(t) + \frac{W_i}{\pi} \int_{-\infty}^{\infty} i_i(\omega) \frac{V_i}{U_i} e^{i\omega t} d\omega \right]$$  (70)
It is apparent that, LOI takes place in a transient field when the condition $B_{zv}(t)=B_{zv}^{'}(t)$ is met, shown in Eq. (71) indicating that the analytical expression can theoretically be formulated by finding the root of the condition.

$$\sum_{i=1}^{\infty} M_i \cdot \text{int}_i F_i \cdot (W_i-W_i') \int_{-\infty}^{\infty} i_0(\omega) \frac{V_i}{U_i} e^{-j\omega t} d\omega = 0$$  (71)

Nevertheless, the deduction of the analytical expression of LOI in a transient field is formidable, in light of the difficulty in giving the expression for the infinite integral within Eqs. (57) and (70). To overcome this, according to [133], the LOI for a transient field can be readily acquired by superimposing the LOIs occurring for all the frequency components within the transient excitation. As a result, the analytical expression of LOI for PEC can be written as:

$$T_{tot} = \sum_{k=1}^{n} T_{tot}(\omega_k)$$

$$= \sum_{k=1}^{n} \frac{1}{\omega_k} \tan^{-1} \left[ \frac{2\sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_i F_i \cdot \left(1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) (W_i-W_i')}{\omega_k \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_i F_i \cdot \left(1 - \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} \right) (W_i-W_i')} \right]$$  (72)

Where, $k$ denotes $k^{th}$ harmonic within the transient excitation; $n$ is the highest order of the harmonic.

**6.2 Analytical formulation of LOI with sensors at arbitrary locations**

This section gives the analytical expression of LOI with the magnetic field sensor placed at a distance from the axi-symmetric axis of the driver coil.
6.2.1 Field formulation

Suppose the Hall sensor is moved from the centre of the driver coil to a location offset with respect to the axi-symmetric axis of the driver coil. Therefore, Figure 32 is modified and the new model is presented in Figure 43. The offset of the Hall sensor is set as \( w \neq 0 \) and \((w + r_0) \leq r_1\). The top view of the modified model is shown in Figure 44.

As can be seen from Figure 44, the trigonometric relation between \( l \), \( w \), \( \varphi \) and \( r_0 \) can be written as:

\[
l = \sqrt{r^2 + w^2 - 2rw\cos \varphi}
\]  

(73)

Figure 43. A 2D axi-symmetric eddy current model with the Hall sensor placed in a distance from the axi-symmetric axis of the driver coil
Based on Eqs. (37) and (38), the net magnetic field signal from the Hall sensor with offset is formulated as:

\[
B_{\omega}(\omega)|_{w=0} = \frac{1}{\pi r_0^2 (c_z - c_i)} \iiint B_z(l, z, \omega) dv
\]

\[
= \frac{2}{\pi r_0^2 (c_z - c_i)} \int_0^{2\pi} \int_0^z \int_0^\infty \left[ B_z^{(1)}(l, z, \omega) + B_z^{(2)}(l, z, \omega) \right] r dr d\varphi dz
\]

\[
= \frac{2\mu_0 i_0(\omega)}{h^2 \pi r_0^3 c} \sum_{i=1}^{\infty} \int_0^\pi \int_0^\infty J_0(a l) \chi(a r_1, a r_2) a_i^2 J_0^2(a h) \cdot \text{int}_F \cdot \left(1 + \frac{V_i}{U_i} W_i\right) r dr d\varphi
\]

In consideration of \( l \), Eq. (74) is rewritten as:

\[
B_{\omega}(\omega)|_{w=0} = i_0(\omega) \sum_{i=1}^{\infty} M_i^r \cdot \text{int}_F \cdot \left(1 + \frac{V_i}{U_i} W_i\right) \cdot \text{int}_J
\]

\[
M_i^r = \frac{2\mu_0 \chi(a r_1, a r_2)}{\pi r_0^2 c h^2 a_i^2 J_0^2(a h)}
\]

\[
\text{int}_J = \int_0^\pi \int_0^\infty J_0(a, \sqrt{r^2 + w^2 - 2rw \cos \varphi}) dr d\varphi
\]
Eq. (75) is actually the generalised analytical expression of the net magnetic field signals from the solid-state magnetic field sensors for SFEC inspection of a stratified conductor. Eq. (55) is the specific case where the location of the sensor is at the centre of the driver coil (w=0). The generalised analytical expression of the net magnetic field signals from solid-state magnetic field sensor for PEC can thus be formulated as:

\[
B_\Sigma(t)|_{w=0} = \sum_{i=1}^{\infty} M_i \cdot \int_{-\infty}^{\infty} i_0(t) \frac{W}{\pi} e^{j\omega t} d\omega \cdot \int_{-\infty}^{\infty} J_i 
\]  

(76)

If the condition for PMF, that the dimension of the sensor is negligible \((c_2=c_1=z; r_0=0)\), applies, Eqs. (75) and (76) are modified as:

\[
\begin{bmatrix}
B_z(\omega, w, z)|_{w=0} = i_0(\omega) \sum_{i=1}^{\infty} m_i \cdot F(a_i, z_1, a_i, z_2, a_i, z) \cdot \left[ 1 + \frac{V}{U_1} P_i \right] \\
B_z(t, w, z)|_{w=0} = \sum_{i=1}^{\infty} m_i \cdot F(a_i, z_1, a_i, z_2, a_i, z) \cdot \left[ i_0(t) + \frac{P}{\pi} \int_{-\infty}^{\infty} i_0(\omega) \frac{V}{U_1} e^{j\omega t} d\omega \right] \\
m_i = \frac{\mu_0 J_0(a_i w) \chi(a_i r_1, a_i r_2)}{a_i [h J_0(a_i h)]^2} 
\end{bmatrix} 
\]  

(77)

Where, the coefficient \(P_i\) is expressed as:

\[
P_i = \begin{cases}
e^{-a_i (z_2 + z)} & z \geq z_2 \\
e^{-a_i (z_1)} - e^{-a_i (z)} & z_2 \geq z \geq z_1 \\
\frac{e^{-a_i z} - e^{-a_i (2z - z_2)}}{2} & z \geq z \geq z_1 \\
e^{-2a_i z} & z_1 \geq z \geq 0
\end{cases}
\]  

(78)

**6.2.2 Generalised expressions of LOI**

Regarding the derivation of the generalised expressions of LOI concerning time-harmonic and transient fields for SFEC and PEC, respectively, the non-magnetic conductive half-space is employed, whilst the probe consists of the driver coil and the
Hall sensor placed with offset to the axi-symmetric axis of the driver coil. Compared to Figure 42, the modified model is shown in Figure 45.

After separating the real and imaginary terms within Eq. (75), the magnetic field signals from the offset Hall sensor can be rewritten as:

\[
B_{z}(\omega)|_{r=0} = \Re \left[ B_{z}(\omega)|_{r=0} \right] + j \Im \left[ B_{z}(\omega)|_{r=0} \right]
\]

\[
\Re \left[ B_{z}(\omega)|_{r=0} \right] = i_0(\omega) \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left[ 1 + \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) W_i \right] \cdot \text{int}_J
\]

\[
\Im \left[ B_{z}(\omega)|_{r=0} \right] = \frac{2i_0(\omega)}{\omega \mu_0 \sigma} \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left[ 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right] W_i \cdot \text{int}_J
\]

**Figure 45.** A 2D axi-symmetric eddy current model involving a cylindrical coil, a non-magnetic conductive half-space and a Hall sensor placed in a distance from the axi-symmetric axis of the driver coil.
Note that the term $int_J_i$ is independent of the lift-off variation. Consequently, the formulation of the expression for LOI becomes straightforward. The generalised analytical expression of LOI for SFEC is written as:

$$
T_{lo} = \frac{1}{\omega} \tan^{-1} \left[ \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) (W_i - W_i' \text{int}_J_i) }{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) (W_i - W_i' \text{int}_J_i)} \right]
$$

(80)

Thus the individual expressions for different Hall sensor vertical locations are given as follows:

$$
T_{lo} \big|_{z_2 < z_2 > z_1} = \frac{1}{\omega} \tan^{-1} \left[ \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) (1 - e^{-2a_i/\omega}) e^{-a_i(z_1+z_2)} \cdot \text{int}_J_i }{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) (1 - e^{-2a_i/\omega}) e^{-a_i(z_1+z_2)} \cdot \text{int}_J_i} \right]
$$

(81)

$$
T_{lo} \big|_{z_2 > z_2 > z_1} = \frac{1}{\omega} \tan^{-1} \left[ \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) (1 - e^{-2a_i/\omega}) e^{-a_i(z_1+z_2)} \cdot \text{int}_J_i }{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) (1 - e^{-2a_i/\omega}) e^{-a_i(z_1+z_2)} \cdot \text{int}_J_i} \right]
$$

(82)

$$
T_{lo} \big|_{z_2 > z_1 > z_1} = \frac{1}{\omega} \tan^{-1} \left[ \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F_i \cdot \left( 1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) L_i \cdot \text{int}_J_i }{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F_i \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1 \right) L_i \cdot \text{int}_J_i} \right]
$$

(83)

Regarding to the expression of LOI for transient field, it can be formulated based on the Eq. (72):
\[
T_{loi} = \sum_{k=1}^{n} T_{loi}(\omega_k)
= \sum_{k=1}^{n} \frac{1}{\omega_k} \tan^{-1} \left[ \frac{2 \sum_{i=1}^{n} a_i^2 M_i \cdot \text{int}_F \cdot \left(1 - \frac{\sqrt{R + a_i^2}}{a_i \sqrt{2}}\right)(W_i - W_i) \text{int}_J}{\omega_k \mu_0 \sigma \sum_{i=1}^{n} M_i \cdot \text{int}_F \cdot \left(\frac{a_i \sqrt{2}}{\sqrt{R + a_i^2}} - 1\right)(W_i - W_i) \text{int}_J} \right]
\]

By analysing Eqs. (80) - (84), it can be found that the LOI is highly dependent on the configuration of the EC system in terms of coil dimension \((r_1, r_2, z_2 - z_1)\), driver coil position \((z_1)\), location of the magnetic field sensor \((w)\), the dimension of the sensor \((c_1, c_2, r_0)\) and the electrical properties \((\mu, \sigma)\) of the specimens under evaluation.

### 6.3 Characteristics of LOI

Before introducing the inverse scheme, it is imperative to gain an insight into LOI based on the deduced analytical expression of it. The following investigations were carried out via the analytical expression of LOI for transient field i.e. PEC.

A series of simulations using ETREE were conducted. The model adopted in the simulations is shown in Figure 43. The excitation current is shown in Figure 46. The probe was modelled, based on a practical probe and its parameters are listed in Table 14. A conductive half-space with conductivity \(\sigma = 26.6\) MS/m and relative permeability \(\mu_r = 1\) was introduced to represent an Aluminium alloy commonly used in aircraft structures. The Hall sensor (SS495A) was taken into account in the model. The distance \((c_1)\) between the bottom surface of the sensing element and the upper surface of the sample was 0.5 mm. The increment of the lift-off varied from 0 mm to infinity.
Figure 46. The PEC excitation current

Table 14. Parameters of the probe

<table>
<thead>
<tr>
<th>Coil's Outside Radius</th>
<th>Coil's Inside Radius</th>
<th>Coil's height</th>
<th>Number of turns</th>
<th>Design Lift-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_2$ / mm</td>
<td>$r_1$ / mm</td>
<td>$z_2-z_1$ / mm</td>
<td>$N$</td>
<td>$z_1-c_1$ / mm</td>
</tr>
<tr>
<td>9.2</td>
<td>4.8</td>
<td>8.2</td>
<td>230</td>
<td>1</td>
</tr>
</tbody>
</table>

The pulse repetition frequency (PRF) of the excitation current was set at 100 Hz. The maximum amplitude was 500 mA. The sampling frequency was 600 kHz, and the number of points was 6001.

6.3.1 LOI point or LOI range

The predicted signals for four cases of lift-off increments are shown in Figure 47(a). This shows that it is difficult to observe LOIs within the time range. Further investigation gives no results in finding the root of each of the two signals. Therefore, it is concluded when using Hall sensors, whose sensitivity is frequency-independent within its operation frequency range, that LOI does not occur in the transient signals.
Figure 47. (a) The predicted magnetic field signals from the Hall sensor with respect to different lift-off increments; and (b) their first-order derivatives against time along with the zoom-in figure within time range from 0.13 ms to 0.15 ms

However, if we take the first-order derivative of the magnetic field signals against time, the LOI can be found for each two lift-off increments in Figure 47(b). The time derivative of the signal from the Hall sensor is equivalent to the voltage output of a similar pickup coil, which has sensitivity dependent on the frequency. Therefore, it is noteworthy that LOI happens among the signals depicting the variation of magnetic field with respect to the time in terms of voltage, whilst the magnetic field measurement does not exhibit an LOI.

Figure 47(b) also illustrates that not all the signals intersect at a particular point. For variable lift-off, multiple LOI points can be found. These points define a range in which the LOI changes as the lift-off varies. As a result, for measurements with more than two lift-offs the intersections amongst signals show a greater LOI range than a LOI point.
6.3.2 LOI range vs. Hall position

The previous investigation shows that the LOI range takes place within the first-order derivatives of magnetic field signals acquired from the Hall sensor when lift-off varies. It is also interesting to find out the characteristics of the LOI range with respect to different Hall sensor positions, which is beneficial to the study of variation of LOI range with different configurations of eddy current probes e.g. magnetic field sensor array.

By setting \( w \), which represents the radial location of the Hall sensor in Eq. (81), the LOI range against Hall position can be analysed. Two parameters of the LOI range are investigated, which are written as:

\[
\begin{align*}
\text{Width} &= \max(T_{\text{LoI}}) - \min(T_{\text{LoI}}) \\
\text{Centre} &= \frac{[\max(T_{\text{LoI}}) + \min(T_{\text{LoI}})]}{2}
\end{align*}
\]  

Suppose the location of the Hall sensor varies from the 0% to 90% inner radius of the excitation coil. The variations of LOI range width and centre against different Hall positions are exhibited in Figure 48.

![Figure 48](image_url)

(a) LOI range width and (b) centre vs. Hall position
Figure 48 shows that the LOI range width increases as the offset of Hall position with respect to the centre of the excitation coil, in contrast to which the LOI range centre generally drops. Interestingly in particular from Figure 48(b), a trough can be located at around 85% inner radius where it is implied that the LOI range centre reaches a minimum. This could be caused by the computation error, which results from finite number of expansions used in the calculation.

6.3.3 LOI range vs. conductivity of the sample

From Eqs. (81) and (84), it can be found that the LOI is closely related to the properties of the samples under inspection. The LOI range against sample conductivity is analysed based on simulation. The conductivity of the sample varied from 50% to 200% of the original value $\sigma_0$, i.e. 26.6 MS/m (equivalently 23% IACS – 92% IACS). The simulation results are shown in Figure 49.

As can be seen from Figure 49, the LOI range width and centre monotonically increase when the conductivity of the sample increases. With the predefined lift offs adopted in measurements, the trajectory representing the relationship of LOI range width and centre with the variation of conductivity can be used to inversely acquire the practical conductivity of the specimen under inspection, by fitting the measured value into the predicted trajectory.
From the theoretical investigation of the LOI range presented above, it is noticeable that the features of LOI range, i.e. width and centre vary with different configurations of PEC system: positions of magnetic sensors and conductivity of the sample. The results direct the setup of the PEC system. For example, to improve the signal to noise ratio in practical experiments, it is imperative to minimise the LOI range. From the results in Figure 48, it can be seen that the optimal position of the Hall sensor is at the centre of coil whilst the variation of the range centre is in the order of 1μs. In addition, the range centre also shows a much higher sensitivity to the variation of conductivity of the specimen than range width, which is shown in Figure 49. Since LOI has exhibited its sensitivity to the conductors under PEC inspection and independency of the lift-off variation, it is possible to evaluate the conductivity of the sample by using LOI, when the lift-off of the probe is unknown.

6.4 Inverse scheme using LOIs derived from introducing two lift-off variations

6.4.1 Theoretical background and implementation

During actual measurements, the lift off and conductivity of the sample are often unknown, e.g. in evaluation of coated pipes where the thickness of the non-metallic coating and conductivity caused by pipe materials and defects are required. The problem could be resolved by building relationships of magnetic field signals as a function of lift-off and conductivity based on a large number of experiments or simulations, and iteratively attempting to fit the predicted signals for lift-off and conductivity in the database with the actual signals in conjunction with minimising the least-squared error. It is apparent that, the inverse process can take a long time to converge at the optimal results.

In an effort to mitigate the problem, the LOI is employed as an additional feature to evaluate the conductivity of the sample, which can be taken as a conductive half-space. Since the LOI for PEC is actually the superimposition of the LOIs of all its harmonic components. Here, Eq. (80) is employed in the inverse scheme. Suppose that the Hall
sensor is positioned at the centre of the excitation coil and under the driver coil, Eq. (67) is re-written as:

\[
\theta = \tan(\omega T_{lo}) = \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left( \frac{1 - \sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) \left( 1 - e^{-2a_i \omega T_{lo}} \right) e^{-\omega T_{lo} \left( c_i + c_2 \right)}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2} - 1} \right) \left( 1 - e^{-2a_i \omega T_{lo}} \right) e^{-\omega T_{lo} \left( c_i + c_2 \right)}}
\]  

(86)

From Eq. (86) it is found that the unknowns in the problem are \( c_1 \) and \( \sigma \). In order to resolve the unknowns, two equations for them need to be written. As a result, two lift-off variations i.e. \( lo_1 \) and \( lo_2 \) are defined. Consequently, deriving the unknowns, i.e. lift-off \( c_1 \) and conductivity \( \sigma \), is equivalent to finding the roots of the equations listed below:

\[
\begin{align*}
\theta_1 &= \tan(\omega T_{lo_1}) = \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left( \frac{1 - \sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) \left( 1 - e^{-2a_i \omega T_{lo_1}} \right) e^{-\omega T_{lo_1} \left( c_i + c_2 \right)}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2} - 1} \right) \left( 1 - e^{-2a_i \omega T_{lo_1}} \right) e^{-\omega T_{lo_1} \left( c_i + c_2 \right)}} \\
\theta_2 &= \tan(\omega T_{lo_2}) = \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left( \frac{1 - \sqrt{R_i + a_i^2}}{a_i \sqrt{2}} \right) \left( 1 - e^{-2a_i \omega T_{lo_2}} \right) e^{-\omega T_{lo_2} \left( c_i + c_2 \right)}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left( \frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2} - 1} \right) \left( 1 - e^{-2a_i \omega T_{lo_2}} \right) e^{-\omega T_{lo_2} \left( c_i + c_2 \right)}}
\end{align*}
\]  

(87)

However, it is difficult to solve Eq. (87) directly. Alternatively, look-up tables can be built, depicting two databases of LOI individually, for the two lift-off variations with respect to different lift-off against conductivity of the thick conductor. With two lift-off variations predefined, the values of the unknowns can be obtained inversely using the look-up tables in conjunction with the curve interpolation which makes the LOI databases continuous and cover all the LOIs. Figure 50 shows the schematic illustration of the inverse scheme, which is outlined below:
Step 1, to establish databases \( (T_{loi}, c_1, \sigma) \) i.e. look-up tables depicting the LOI vs. lift-off against conductivity via ETREE modelling respectively for two predefined lift-off variations;

Step 2, to derive two subspaces \((c_1, \sigma)\) for the two LOIs, which occur when the variation of lift-off takes place. The subspace defines the relationship between two unknowns i.e. \( c_1 \) and \( \sigma \) which result in the same LOI;

Step 3, to find the solutions to the two unknowns by finding the intersection of the two subspaces in conjunction with the curve interpolation.

### 6.4.2 Verification of the inverse scheme using FEM

In order to assess the effectiveness and accuracy of the proposed inverse scheme, a test magnetic field signal when the probe is placed over a coated conductive half-space was adopted with the lift-off and conductivity unknown. Then the other two signals were acquired by deliberately increasing the lift-off in increments of +0.5 mm and +1 mm. By comparing the first-order derivatives of the signals for each increment with the first-order derivative of the test signal, two LOIs were found at 0.136 ms and 0.138 ms
for lift-off increments of +0.5 mm and +1 mm, respectively. Note that the test signals were obtained from FE simulations.

To inversely obtain the lift-off of the PEC probe and the conductivity of the half-space under inspection, firstly, by applying ETREE modelling, two LOI databases were established showing the lift-off-and-conductivity-dependent LOIs, which were acquired by comparing the first-order derivative of PEC signal for each lift-off with that for the lift-off with predefined increment such as +0.5 mm and +1 mm. The two LOI databases are shown in Figure 51.

Figure 51. Databases showing LOI vs. lift-off against conductivity for lift-off variation of (a) 0.5 mm and (b) 1 mm

The two LOIs were looked up in the database to derive the subspaces \((c_1, \sigma)\), which are illustrated in Figure 52(a). Figure 52(b) shows the deviation of the first subspace for LOI\(_1\) against the second subspace for LOI\(_2\). Although the two subspaces for every LOI look quite similar to Figure 52(a), further analysis in finding the joint point of the subspaces for two LOIs gives a \((c_1, \sigma)\) pair at the joint point which is \((\sigma=33.97 \text{ MS/m, } c_1=5.51 \text{ mm})\). The approximated lift-off and conductivity has good agreement with the true value: \(\sigma=34 \text{ MS/m}\) and \(c_1=5.5 \text{ mm}\). The relative errors are less than 1%.
In summary, the evaluation of conductivity of a conductor ($\sigma$) can be implemented using an inverse scheme in conjunction with the acquisition of LOIs for two lift-off increments ($lo_1$, $lo_2$), regardless of the distance between PEC probe and the surface of the conductor ($c_1$). The lift-off increments can be provided by real measurement, although the original lift-off is unknown. In other words, the original lift-off and sample conductivity can be derived using an inverse scheme through multiple measurements of LOI points and theoretical computation. It can be found from the verification of the inverse process that despite the unknown lift-off, the conductivity of the sample is still accurately estimated based on the characteristics of LOI.

### 6.5 Inverse scheme using LOIs based on measurement with magnetic field camera

#### 6.5.1 Theoretical background and implementation

The essence of the previous inverse scheme lies in the introduction of two lift-off variations so that two equations can be formulated and solved for the unknowns viz. the lift-off of the probe ($c_1$) and the conductivity of the conductive half-space ($\sigma$). From this, an alternative inverse scheme for solving the same problem is proposed by using two magnetic field sensors and employing only one lift-off variation in an effort to adopt the LOIs acquired within the field signals from these two sensors.
The inverse scheme applies with the condition that lift-off variation $lo$ used to acquire LOI and other parameters are predefined beforehand. As a result, to obtain the solutions to those two unknown variables in Eq. (80), two equations need to be set up. Let $L_1$ and $L_2$ denote the individual radial locations of two sensing elements of the magnetic field camera, thus the two equations can be formulated as:

\[
\begin{align*}
\theta_1 &= \tan(\omega T_{\text{lon1}}) = \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left(1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}}\right) (W_i - W_i') \text{int}_J_{i1}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left(\frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1\right) (W_i - W_i') \text{int}_J_{i1}} \\
\theta_2 &= \tan(\omega T_{\text{lon2}}) = \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i' \cdot \text{int}_F' \cdot \left(1 - \frac{\sqrt{R_i + a_i^2}}{a_i \sqrt{2}}\right) (W_i - W_i') \text{int}_J_{i2}}{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left(\frac{a_i \sqrt{2}}{\sqrt{R_i + a_i^2}} - 1\right) (W_i - W_i') \text{int}_J_{i2}}
\end{align*}
\]

(88)

Where,

\[
\left\{ 
\begin{aligned}
\text{int}_J_{i1} &= \int_0^\pi \int_0^{r_0} r J_1 \left(a_i \sqrt{r^2 + L_1^2 - 2rL_1 \cos \varphi}\right) dr d\varphi \\
\text{int}_J_{i2} &= \int_0^\pi \int_0^{r_0} r J_2 \left(a_i \sqrt{r^2 + L_2^2 - 2rL_2 \cos \varphi}\right) dr d\varphi
\end{aligned}
\right.
\]

(89)

Here, a look-up-table is still employed to solve the equations indirectly since it is difficult to derive the expressions of the two unknowns derived from Eq. (88). Within the tables or databases, the two LOIs for the two sensing elements with respect to different lift-offs against conductivities of the thick conductor are built using the ETREE models mentioned previously. Since the lift-off variation is predefined, the values of the unknowns can be inversely obtained by looking them up in the databases in conjunction with the curve interpolation. Figure 53 presents the schematic illustration of the inverse scheme, which is outlined below:
Step 1, to establish the database \((T_{loi}, c_1, \sigma)\) which depicts relationships of the LOI for each sensing element vs. lift-off against conductivity via ETREE modelling for the predefined lift-off variation;

Step 2, to derive two curves i.e. subspaces \((c_1, \sigma)\) showing the relationship between lift-off and conductivity for individual LOIs acquired from practical tests using multiple measurements under known lift-off variation, from which the relationship between two unknowns i.e. \(c_1\) and \(\sigma\) that result in the same LOI becomes explicit;

Step 3, to find the solutions to the two unknowns by finding the intersection of the two curves in conjunction with the interpolation.

\[\text{Database \((T_{loi1}, c_{lo1n}, \sigma_{lo1n})\) built via ETREE with a sensor at \(L_1\)}\]

\[\text{Database \((T_{loi2}, c_{lo2n}, \sigma_{lo2n})\) built via ETREE with a sensor at \(L_2\)}\]

\[\text{Find Intersection}\]

\[\text{Subspace \((c_{lo1n}, \sigma_{lo1n})\)}\]

\[\text{Find Intersection}\]

\[\text{Solution \((c_{lo}, \sigma_{lo})\)}\]

\[\text{Measured LOI, } T_{loi, j} \text{ from a sensor at } L_j\]

\[\text{Measured LOI, } T_{loi, j} \text{ from a sensor at } L_j\]

**Figure 53. Illustration of the inverse scheme using two sensors and one lift-off**

6.5.2 Verification of the inverse scheme using measurement with magnetic field camera

In order to evaluate the proposed inverse scheme, a series of experiments have been conducted to obtain the PEC signals for extraction of LOIs, which are subsequently used to estimate the lift-off and the conductivity of a conductive slab. The corresponding model of ETREE is shown in Figure 54.
Figure 54. ETREE model comprising of a driver coil, two Hall sensors and a conductive half-space

In such a case, the lift-off of the probe is referred as \( z_j \) in lieu of \( c_1 \). Considering the condition that \( c_2 > c_j > z_2 > z_1 \), the databases \((T_{\text{lo1}}, z_1, \sigma)\) and \((T_{\text{lo2}}, z_1, \sigma)\) respectively for the magnetic field sensor \( a_1 \) and \( a_2 \) are established based on the equations:

\[
T_{\text{lo1}} = \sum_{i=1}^{n} \frac{1}{\omega_k} \tan^{-1} \left( \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left(1 - \frac{\sqrt{R + a_i^2}}{a_i \sqrt{2}} \right) \left(1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)} \right)
\frac{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left(1 - \frac{a_i \sqrt{2}}{\sqrt{R + a_i^2}} - 1 \right) \left(1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)} \right)
\]

\[
T_{\text{lo2}} = \sum_{i=1}^{n} \frac{1}{\omega_k} \tan^{-1} \left( \frac{2 \sum_{i=1}^{\infty} a_i^2 M_i \cdot \text{int}_F \cdot \left(1 - \frac{\sqrt{R + a_i^2}}{a_i \sqrt{2}} \right) \left(1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)} \right)
\frac{\omega \mu_0 \sigma \sum_{i=1}^{\infty} M_i \cdot \text{int}_F \cdot \left(1 - \frac{a_i \sqrt{2}}{\sqrt{R + a_i^2}} - 1 \right) \left(1 - e^{-2a_i \omega_0} \right) e^{-a_i (z_1 + z_2)} \right)
\]

\[
\text{int}_J_{12} = \int_0^\pi \int_0^\varphi \rho J_0 \left(a_i \sqrt{r^2 + w^2 - 2rw \cos \varphi} \right) dr d\varphi
\]

(90)

6.5.2.1 Experimental setup

Figure 55(a) presents the schematic illustration of the PEC system, which consists of the PEC probe and modules for signal generation, conditioning and acquisition. A close-up
picture of the PEC probe is shown in Figure 55(b). The probe comprises an excitation coil and the magnetic field camera which contains an array of 64 (8 × 8) Hall sensors (Samsung HE12AF1U12, 320 mV for a 500 G magnetic field). The dimension of the Hall sensing element is 0.53 × 0.28 mm². The entire array covers the area of 27 mm², and was placed over the windings of the driver coil with the distance of 9.85 mm from the surface of the slab (c1=9.85 mm). The parameters of the excitation coil are listed in Table 1.

![Figure 55. (a) The schematic illustration of PEC system; (b) The PEC probe used in the experiments](image)

<table>
<thead>
<tr>
<th>Outer Diameter $2r_2$ / mm</th>
<th>Inner Diameter $2r_1$ / mm</th>
<th>Height $z_2-z_1$ / mm</th>
<th>Design lift-off $z_1$ / mm</th>
<th>Number of turns $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>21.90</td>
<td>16.83</td>
<td>8.50</td>
<td>0.64</td>
<td>1700</td>
</tr>
</tbody>
</table>

The excitation current is shown in Figure 56 along with the illustration of the predicted current, which was employed in the ETREE modelling to establish the database i.e. the distribution of LOI against various conductivities and lift-offs. Figure 57 shows the 3D plot of the distribution of the magnetic field generated by the excitation coil when it was placed in air, acquired from the magnetic field camera. Although the field profile is not smooth due to the low spatial resolution of the camera (3.5 mm), two Hall elements ($a_1$, $a_2$ in Figure 54) of the magnetic field camera placed over the driver coil were selected under the criteria that the discrepancy between the theory and experiment is minimum with regard to the coil field strength: $a_1$ i.e. the central Hall element is located at the centre of the coil ($w=0$ mm) while the other $a_2$ i.e. the offset Hall element is 3.5 mm away from the axi-symmetric axis of the coil ($w=3.5$ mm). The PEC signals from the
two Hall elements obtained via ETREE modelling and practical measurement when the probe was placed in air are shown in Figure 58. Good agreement can be found between theory and experiment, which ensures the validity of the established databases used in the inverse process.

Figure 56. The measured and predicted excitation current

Figure 57. The distribution of half of the coil field obtained using the magnetic field camera
During the course of the experiment, the PEC probe was firstly placed on the surface of a thick conductive, nonmagnetic slab (as a conductive half-space) with a thickness of over 100 mm. The PEC responses to the specimen from the two Hall elements were acquired. Following that, a wooden plate was placed between the excitation coil and the surface of the half-space in order to introduce the lift-off variation. The thickness of the wooden plate is 1.58 mm ($l_o=1.58$ mm). Figure 59 illustrates the PEC signals obtained from the individual Hall element with and without the presence of the lift-off variation, from which the LOI for each Hall element was acquired by finding the intersection point of the first-order derivatives of the two PEC signals against time ($dB_z/dt$). Further processing gave the LOIs at 0.337 ms and 0.342 ms for the central Hall element and the offset Hall element, respectively, which also proves via experiments that the LOI is dependent on the sensor location where the magnetic signals are obtained.
Figure 59. The PEC signals from the central Hall element and the offset Hall element with and without the lift-off variation

Figure 60 presents the two LOIs obtained from the practical tests (shown as slices in 3D representation) and the 3D distributions of predicted LOIs against different conductivities and lift-offs for each individual sensor, which were derived from the simulations using ETREE modelling.

(a) (b)

Figure 60. The distribution of LOI vs. $\sigma$ and $z_l$ for the two Hall elements: (a) the centre Hall element; (b) the offset Hall element

The intersections between the two LOIs and the distributions give two curves (subspaces) depicting the combinations of $\sigma$ and $z_l$, which result in the same measured LOIs. The curves are illustrated in Figure 61(a). As shown in Figure 61(b), the subtraction of the two curves subsequently gives the estimation of the lift-off and the conductivity of the half-space: $z_l = 2.41$ mm; $\sigma = 27.64$ MSm which have a discrepancy
of 8.56% and 5.02% respectively, against the true values of: $z_l=2.22 \text{ mm}$; $\sigma=29.10 \text{ MSm}$. 

![Graph](image1.png)

(a) The relations between lift-off and $\sigma$ for the two Hall elements

![Graph](image2.png)

(b) Subtraction of the two curves

Figure 61. (a) The two subspaces ($z_l$, $\sigma$) and (b) the difference between the two subspaces

It can be seen that by comparing the approximated lift-off and the conductivity of the nonmagnetic half-space with the practical values, the inverse scheme is able to provide an acceptable estimation regarding the two unknowns of the PEC system during the inspection. Using the magnetic field camera (2D sensor array), the multiple unknowns of the parameters of the PEC system could be derived since the equations for LOIs extracted from the magnetic signals at various points are solvable. However, it should be pointed out that for more than two unknowns, the distribution of LOI is not in 3D but in multiple dimensions, and thus the inverse scheme becomes intricate in terms of computation due to decreasing the dimensionality of the matrices.

6.6 Chapter summary

In light of the advantages of ETREE over traditional analytical modelling using integral expression and TREE method, this chapter shows: (1) the formulation of analytical expression of LOI taking place within the magnetic field signals from solid-state magnetic field sensors with varied lift-off, based on ETREE modelling; (2) the inverse
schemes for evaluation and estimation of the conductivity of a conductive half-space and the lift-off of the EC probe.

Two cases are taken into account during the derivation of the analytical expression of LOI for SFEC and PEC: (1) the magnetic field sensor is placed at the centre of the driver coil; and (2) the sensor is placed at the location with radial offset with respect to the axi-symmetric axis. The first case is actually the specific case when the offset is set as zero in the second case, where the generalised expression of the magnetic field signals from solid-state magnetic field sensors positioned at an arbitrary location above the specimens has been obtained. Following the separation of real and imaginary terms within the expression of magnetic field signals, the time instant when the LOI occurs for SFEC and PEC has been expressed with closed-form equations. It is noted that the LOI is dependent on the configuration of EC inspection systems involving the excitation frequency, locations of the sensors or sensor array, size of the driver coil, and lift-off variation of the probe, etc. It has also been found via theoretical investigation that with multiple lift-off variations, the magnetic field signals do not intersect at a particular point, instead, multiple LOIs are acquired. Therefore, for measurements with more than two lift-off variations the intersections amongst signals show a greater LOI range than a LOI point. The range width and range centre vary with the position of the sensor and the conductivity of the sample.

Since LOI has exhibited sensitivity to the lift-off of the EC probe and the conductivity of the sample in forward theoretical investigation, two inverse schemes based on the LOI have been proposed. These are based on the databases established via forward ETREE modelling and depicting the relationship of LOI with variable probe lift-offs and conductivities of half-spaces. Two LOIs are needed in the inverse process. Regarding the first inverse scheme, the two LOIs are obtained by introducing two lift-off variations during the course of the magnetic field measurement, using a single magnetic field sensor. In contrast, as for the second inverse scheme, the two LOIs are acquired within the magnetic field signals from two sensors whilst only one lift-off
variation is involved. By looking up two LOIs in the established databases (look-up tables), the lift-off of the probe and the conductivity of the thick slab can be estimated.

The two inverse schemes are verified through FE simulations and practical measurement of magnetic field signals by using a magnetic field camera respectively. By comparing the estimated lift-off and conductivity with the real values, it can be seen that the proposed inverse schemes are capable of providing values having good agreement with the true values with acceptable discrepancies.

This work has demonstrated how to apply analytical models in conjunction with experimental tests for inversely retrieving information about sample material and surface geometry or form from SFEC and PEC inspection. More experimental verification will be studied in further work.
CHAPTER 7
CONCLUSION AND FURTHER WORK

In this chapter, the research work presented in the thesis is summarised. The conclusions are derived, where contribution of the work in ENDE is detailed and highlighted. Based on the research outcome, further work is addressed.

7.1 Concluding remarks

This thesis focuses on the theoretical and experimental investigations of ENDE techniques especially SFEC and PEC inspection of multilayered structures in an effort to present: (1) an extensive literature survey on ENDE techniques and the theoretical simulations via numerical and analytical approaches; (2) an intensive evaluation of two commercial FE simulation packages, COMSOL and ANSOFT MAXWELL EM for ENDE modelling via a series of case studies involving MFL and RFEC in a bid to select the preferred package to provide the FEA results for verification of ETREE modelling of SFEC and PEC; (3) a fast and accurate analytical model i.e. ETREE modelling for SFEC and PEC inspection of multilayered specimens, along with the implementation of FE simulation in the same modelling scenario, and the comparison between theory (ETREE modelling and FEA) and experiment; (4) a derivation of the analytical expression of LOI occurring in SFEC and PEC via ETREE modelling, which allows the characterisation of LOI with respect to the configurations of inspection systems; (5) two inverse schemes for obtaining the conductivity of a thick conductive sample and estimating the lift-off of an EC probe, based on the database which is established using the analytical expression of LOI derived from ETREE, and contains LOI against various sample conductivities and probe lift-offs.

Based on the research outcome, the major conclusions can be summarised as follows:
1. Two commercial FEM packages i.e. ANSOFT MAXWELL EM and COMSOL have been evaluated via a series of cases studies regarding ENDE forward simulations. COMSOL is found superior to ANSOFT MAXWELL EM in terms of adaptive meshing, flexible mesh conditions, collaboration with MATLAB and capacity of handling moving component. Thus it is preferred for FEA of SFEC and PEC.

2. The analytical modelling method for SFEC and PEC, namely ETREE has been established to predicting magnetic field signals from solid-state magnetic field sensors during the inspection of multilayered structures, while taking sensor dimension into account. ETREE has been found advantageous over FEM in terms of 5% higher accuracy with respect to experimental results and 99% faster computation time.

3. Based on ETREE, the analytical expression of LOI occurring in SFEC and PEC has been formulated. The characteristics of LOI have been investigated. Since LOI is found dependent of the lift-off of probes and the conductivity of samples, it is employed in the proposed inverse schemes for estimating the lift-off of probes and conductivity of samples. ETREE is used to establish the database depicting the relation between LOI and different combinations of lift-off and conductivity. The inverse schemes have been verified via FEM and experiment. The estimated lift-off and conductivity by using the inverse schemes have good agreement with the real values.

Each conclusion is elaborated in the following sections.

7.1.1 ETREE modelling of SFEC and PEC inspections on multilayered structures

Although the advantages of FEA have been highlighted in the previous study, its drawbacks are noted. These include time-consuming computation and trade-off between computation time and accuracy. The drawbacks hinder the application of FEA to efficient simulations for SFEC and PEC in terms of the fast and accurate prediction of magnetic field signals.
The ETREE modelling has been proposed based on the previous analytical modelling involving an integral expression method and TREE method. The modelling approach avails the prediction of magnetic field signals from solid-state magnetic field sensors in lieu of impedance signals of induction coils. It is applicable in collaboration with experimental investigations of SFEC/PEC inspection of multilayered structures, which employ magnetic field sensors to enhance the detectability, the spatial resolution, etc. At the same time, it inherits the advantages of TREE over the integral expression method, which are (1) the ease of computation due to the replacement of infinite integrals with a series of eigenfunction expansions; (2) more feasibility in adjusting the computational error by choosing the adequate number of expansions and eigenvalues. In the model, the location of the magnetic field sensor can be arbitrary since the formulation of the analytical expression also takes the offset of the sensor with respect to the symmetric axis of the driver coil into account.

In order to verify ETREE for SFEC/PEC, FEM is employed. Firstly, the two commercial FEM packages, COMSOL and ANSOFT MAXWELL EM are evaluated via case studies of FE simulations for ENDE including MFL and RFEC. The modelling involves (1) 3D static FEA; (2) transient FEA in conjunction with mechanical translation of components within models; (3) the modelling of remote field concerning the large-dimension components within models. It has been found that COMSOL is more feasible for FE simulations for ENDE, and preferred in the study.

The simulations of the SFEC and PEC inspection of two types of multilayered structures have been conducted via ETREE and FEM (implemented in COMSOL). The magnetic field signals as a function of not only frequency for SFEC but also time for PEC have been predicted. The comparison between theory and experiment shows good agreement between the predicted signals and the measurement results. In consideration of simulation accuracy and time, ETREE modelling is superior to FEM in terms of higher computation accuracy and faster calculation time. This is due to the intrinsic
characteristics of ETREE in that: (1) the expression is closed-form and mesh-independent; (2) most coefficients in the expression are independent of frequency.

The proposed ETREE modelling for SFEC and PEC benefits the inverse process for rapidly acquiring the parameters of the inspection system from the test signals. It is also noticed that based on current work, the ETREE modelling for SFEC/PEC inspection of specimens with defects could be achievable as long as the eigenvalues describing the flawed regions within the specimens are computed.

7.1.2 Inverse schemes using LOI

Following the derivation of the analytical expressions of magnetic field signals from magnetic field sensors in SFEC and PEC, the formulation of closed-form equations of LOI is conducted via ETREE modelling.

From the analytical expression of LOI, the characteristics of LOI and its relationship with the configurations of the inspection systems have been scrutinised. From the theoretical investigation, LOI has been found to exhibit more range characteristics than a single point. The features of the range such as width and centre vary with the system setups involving, for example, the conductivity of the sample and the location of the magnetic field sensor.

Since the relationship of LOI with system setups, such as sample conductivity and probe lift-off is presented explicitly via the analytical expression. Two inverse schemes have been proposed in conjunction with the characteristics of LOI, in order to inversely estimate the conductivity of a thick conductive sample and the lift-off of an EC probe, simultaneously. These parameters are unknown during the inspection, and need evaluation (for conductivity) and compensation (for lift-off).

The essence of the two inverse schemes is to derive the solutions to the two unknowns (conductivity and lift-off) in the two equations depicting the relationship between LOI
and conductivity as well as lift-off, as the LOIs with respect to the two system setups are already known from tests. A look-up table in conjunction with data interpolation is used to obtain the solutions. The first inverse scheme is based on two databases containing the relationship of LOIs with conductivity and lift-off via ETREE modelling by introducing two lift-off increments. The LOIs found in test signals acquired using FE simulations have intersections with the two databases, which results in the two curves giving pairs of lift-off and conductivity which lead to the same LOI. The joint point of the two curves provides the final results for the lift-off and conductivity. Similarly, the second inverse scheme uses the two databases containing the LOIs for two magnetic field sensors at different locations. Two LOIs are acquired from the magnetic field signals obtained in experimental measurement using the magnetic field camera. By looking up these two LOIs in the databases, the estimation of conductivity of the thick sample and lift-off of the probe is achieved.

The current work could be extended to inverse schemes for obtaining the conductivity profiles of specimens and defect characterisation, which is involved in further research, since the two issues could be taken as inverse problems regarding conductivity of samples. The utilisation of multiple sensors (sensor arrays) and magnetic field patterns would help address the problems.

7.2 Further work

Following the research outcomes achieved in this research, the directions of future work are given in terms of ETREE modelling, the inverse process and other relevant issues.

7.2.1 ETREE for EC forward problems involving natural and complex-shaped defects

The current ETREE modelling is applicable to the simulations of SFEC and PEC inspection of layered conductors. However, the modelling for complex-shaped or natural defects has not yet been implemented. The further work will involve the ETREE modelling of SFEC and PEC inspection of conductors with anomalies including (1)
natural defects and (2) complex-shaped (irregular surface shape) defects, in an attempt to expand the application of ETREE to more simulations of EC forward problems.

Based on the previous work, the modelling for natural and complex-shaped defects could be achievable. As can be seen from the formulation of closed-form expression of magnetic field signals using ETREE, besides the coefficients within the expansion series, the eigenvalues are paramount and crucial for establishing the expression and deriving the final solutions. Since the conductors have the defects in the problem domain, the eigenvalues vary compared to those for flawless conductors. It is imperative to obtain the modified eigenvalues. This could be resolved by numeric evaluation of the equation depicting the boundary condition imposed on the edge of the defects, which is related to derivation of roots of the equation comprising Bessel functions and exponential functions. After all of the eigenvalues are obtained, the solution to magnetic field signals from solid-state magnetic field sensors becomes straightforward.

7.2.2 Extension of the inverse schemes with LOIs using sensor arrays

As shown in Chapter 6, the inverse schemes proposed are used for inversely evaluating the lift-off of the EC probe and the conductivity of a conductive half-space whose thickness is much larger than the dimension of the driver coil. The work could be extended to the inverse process for estimating the conductivity of a layered structure which is frequently used in aircrafts. The key to addressing the issue is the separation of the real and imaginary parts of the conductor reflection coefficient within the expression of the magnetic field signals, in an effort to formulate the generalised analytical expression of LOIs.

This further work also involves the inverse assessment of the conductivity profile of the sample under SFEC/PEC inspection and 3D reconstruction of defects within layered structures, since prior to the derivation of the databases the conductivity of the sample against the depth has to be determined. This is concerned with: (1) the determination of
the number of layers; (2) the conductivity of each layer. It can be seen that more variables would be introduced during the inverse process, which results in multi-dimensional governing matrix derived from Eqs. (87) and (88). Consequently, the reduction of the dimensionality of the matrix is essential, which could be addressed by employing magnetic field sensor arrays for acquiring multiple LOIs.

Furthermore, more experimental study and verification of inverse schemes will be conducted using samples from industry such as coated pipes and plane wings.

### 7.2.3 Magnetic field imaging using Magnetically Actuated Micromirrors

Chapters 5 and 6 introduce ETREE modelling for SFEC/PEC inspection with solid-state magnetic field sensors placed at arbitrary positions. It is worth noting that the model can be used to not only predict the magnetic field signals from a sensor array but also provide the magnetic field distribution within a localised region. It could be of great interest to compare theory with experiment regarding the field distribution besides the magnetic field signals at individual positions, which concerns the acquisition of magnetic field profile via experiments, i.e. magnetic field imaging.

In addition to magnetic sensor arrays or magnetic camera using solid-state magnetic sensors e.g. Hall devices or GMR, previous research has shown the functionality of Magnetically Actuated Micromirrors (Papers 4, 9 and 14 in LIST OF PUBLICATIONS) via FE simulations and experiments. It could be adopted in mapping the 2D distribution of magnetic field with higher spatial resolution (in the order of 100μm).

Thus, the further work will involve: (1) magnetic field mapping for ENDE inspections with high spatial resolution for quantification of small defects with geometrical dimension in the order of μm; (2) to experimentally acquire magnetic field distribution during SFEC/PEC inspection of multilayered structures by using Magnetically Actuated Micromirrors; (2) to compare the predicted mapping of magnetic field via ETREE and FEM with measured results. The work would benefit: (1) fast real-time ENDE
inspection of conductive specimens for defect detection and characterisation; (2) inverse schemes in conjunction of ETREE for obtaining the specimen conductivity profile, and 3D defect reconstruction, as proposed in Section 7.2.2.
List of References


94. T. P. Theodoulidis and J. R. Bowler, ‘The truncated region eigenfunction expansion method for the solution of boundary value problems in eddy current


Appendices

A. ETREE modelling code for SFEC inspection of multilayered structures

A.1 Main function

clc;
tic;

% current
freq=albr(:,1);
cur=(1/1000)*albr(:,2);

% Some preliminary definitions and input data%
mm=1.0e-3; MSm=1.0e6; m0=pi^4.0e-7;
% Wire turns and input current
WT=804;
% Inner and outer radii of the coil
r1=((21.35/2)+0.64)*mm; r2=(24.6/2)*mm;
% Bottom (z1) and top (z2) heights of the coil
z1=0.64*mm; z2=z1+(7.87-2*0.64)*mm;
% Truncation limit and number of series terms for the TREE method
h=20*r2; Ns=500;

% Hall dimension
r0_hall=mm*0.908548;
z0_hall=(0.48-(0.68326/3))*mm;
z1_hall=z0_hall+(2*0.68326/3)*mm;
s_hall=pi*(r0_hall^2)*(z1_hall-z0_hall);

J1roots=besselzeros(Ns);
q=J1roots/h;

x1=q*r1;
x2=q*r2;
Jx=1:1:Ns;

% quadrature approx. of bessel integration
for cntj=1:Ns
    Jx(cntj)=quadl(@yy,x1(cntj),x2(cntj),1e-6);
end
bes = h*q.*besselj(0,q*h);
oros = Jx./q./(bes.^2);
% number of layers
nol=2;
cond1=[34.2, 14]*MSm;
% position of layer
pos_layer=1.5*mm;

results=zeros(1,length(freq));
WTD=WT*cur/((r2-r1)*(z2-z1));
coef=m0*WTD;

%------
int_bessel=(1./q).*r0_hall.*besselj(1,r0_hall*q);
int_eqzB_underc=(1./q).*(exp(q*(z1-z1))-exp(q*(z0_hall-z1))-exp(q*(z1-z2))+exp(q*(z0_hall-z2)));
int_eqzB_withinc=2*(z1_hall-z1)-(1./q).*(exp(q*(z1_hall-z2))-exp(q*(z1-z2))-exp(q*(z1-z1_hall))+exp(q*(z1-z1)));
B2=oros.*(int_eqzB_underc+int_eqzB_withinc);
int_eqzA=(1./q).*(exp(-q*(z1_hall+z2))-exp(-q*(z0_hall+z2))-exp(-q*(z1_hall+z1))+exp(-q*(z0_hall+z1)));
%------
if (length(cond1)==nol)&(length(pos_layer)==nol-1)
    warning('Processing %d layers case... 
',nol);
for cnt_freq=1:length(freq)
    warning('Processing %d sample... ',cnt_freq);
    omega=2.0*pi*freq(cnt_freq);
    p=zeros(Ns,nol);
    for cntp=1:length(cond1)
        p(:,cntp)=conj(sqrt(q.*q+i*omega*m0*cond1(cntp))');
    end
    A2_coe=matrixgen(nol,p,q,pos_layer); % Conductor reflection coefficient
    A2=oros.*A2_coe.*int_eqzA;
    results(cnt_freq)=(2*pi/s_hall)*coef(cnt_freq)*sum(int_bessel.*(A2+B2));
end
toc;

A.2 Sub function ‘yy’

function y_fun=yy(x)
y_fun=x.*besselj(1,x);

### A.3 Sub function ‘matrixgen’

```matlab
function A2=matrixgen(layer_no,eigen_layer_all,eigen_air,position_layer)
  
% function: matrixgen(layer_no,eigen_layer_all,eigen_air,position_layer)

% 'eigen_layer_all' should be 'Ns'-by-'layer_no'.

AA2=zeros(1,length(eigen_air));
layer=layer_no;
q=eigen_air;

switch layer

  case 1
    d=1e-3;
    p1=eigen_layer_all(:,1);
    p2=eigen_layer_all(:,1);
    p1=conj(p1');
    p2=conj(p2');

    qp1a=q+p1;
    qp1s=q-p1;
    p1p2a=p1+p2;
    p1p2s=p1-p2;
    ep1d=exp(-2.0*p1*d);

    A2=(qp1s.*p1p2a+qp1a.*p1p2s.*ep1d)./(qp1a.*p1p2a+qp1s.*p1p2s.*ep1d);

  case 2
    d=position_layer;
    p1=eigen_layer_all(:,1);
    p2=eigen_layer_all(:,2);
    p1=conj(p1');
    p2=conj(p2');

    qp1a=q+p1;
    qp1s=q-p1;
    p1p2a=p1+p2;
    p1p2s=p1-p2;
    ep1d=exp(-2.0*p1*d);

    A2=(qp1s.*p1p2a+qp1a.*p1p2s.*ep1d)./(qp1a.*p1p2a+qp1s.*p1p2s.*ep1d);

end
```

157
\[ A2 = \frac{(qp1s \cdot p1p2a + qp1a \cdot p1p2s \cdot ep1d)}{(qp1a \cdot p1p2a + qp1s \cdot p1p2s \cdot ep1d)}; \]

otherwise

\[ \text{inte\_layer} = \text{layer} - 2; \]
\[ \text{unkn} = \text{layer} \cdot 2; \]

% position is minus
\[ \text{position\_layer} = (-1) \cdot \text{position\_layer}; \]
% eigenvalues for air
% base matrix
\[ \text{base\_a} = \begin{bmatrix} 1, 1, -1, -1 \end{bmatrix}; \]
\[ \text{base\_b} = \begin{bmatrix} -1, 1, 1, -1 \end{bmatrix}; \]
% position
\[ \text{position\_layer} = \text{position\_layer} - 1 \cdot \text{position\_layer}; \]
% eigenvalues for air
% base matrix
\[ \text{base\_a} = \begin{bmatrix} 1, 1, -1, -1 \end{bmatrix}; \]
\[ \text{base\_b} = \begin{bmatrix} -1, 1, 1, -1 \end{bmatrix}; \]
% interface
\[ \text{th} = \text{position\_layer}(1:(\text{length(position\_layer)} - 1)); \]
% bottom-layer interface
\[ \text{thh} = \text{position\_layer}(\text{length(position\_layer)}); \]
for \( \text{cnt\_a2} = 1: \text{length(eigen\_air)} \)
\[ \text{eigen\_layer} = \text{eigen\_layer\_all}(\text{cnt\_a2},:); \]
% eigenvalues for internal layers
\[ \text{p} = \text{eigen\_layer}(1:(\text{length(eigen\_layer)} - 1)); \]
% bottom-layer eigenvalues
\[ \text{botto\_p} = \text{eigen\_layer}(\text{length(eigen\_layer)}); \]
% upper 2*n array and bottom 2*n array
\[ \text{upper\_m} = \text{sparse}([1, -1, 1, 1, \text{zeros}(1, \text{unkn} - 3); -1, 1, 1, 1, \text{zeros}(1, \text{unkn} - 3)]) \]
\[ \times \text{sparse}([1, 1, 1, \text{zeros}(1, \text{unkn} - 3); q(\text{cnt\_a2}), p(1), p(1), \text{zeros}(1, \text{unkn} - 3))]; \]
\[ \text{botto\_exp1} = \exp([\text{p} \cdot (\text{layer} - 1), \text{p} \cdot (\text{layer} - 1), \text{botto\_p}] \cdot \text{thh}); \]
\[ \text{botto\_exp2} = [\text{p} \cdot (\text{layer} - 1), \text{p} \cdot (\text{layer} - 1), \text{botto\_p}] \cdot \text{botto\_exp1}; \]
\[ \text{botto\_m1} = \text{sparse}([\text{zeros}(1, \text{unkn} - 3), 1, 1, -1]) \]
\[ \times \text{sparse}([\text{zeros}(1, \text{unkn} - 3), \text{botto\_exp1})]; \]
\[ \text{botto\_m2} = \text{sparse}([\text{zeros}(1, \text{unkn} - 3), -1, 1, 1]) \]
\[ \times \text{sparse}([\text{zeros}(1, \text{unkn} - 3), \text{botto\_exp2})]; \]
\[ \text{botto\_m} = \text{sparse}([\text{botto\_m1}; \text{botto\_m2})]; \]
% initialisation
\[ \text{temp1} = []; \]
% matrix factorisation
\[ \text{for} \; \text{cnt} = 1: \text{inte\_layer} \]
\[ \text{cntt} = 2 \cdot \text{cnt} - 1; \]
\[ \text{expp} = \exp([-\text{p} \cdot (\text{layer} - 1), \text{p} \cdot (\text{cnt}), \text{p} \cdot (\text{cnt} + 1)]) \cdot \text{th}(\text{cnt})]; \]
\[ \text{pexpp} = [\text{p} \cdot (\text{cnt}), \text{p} \cdot (\text{cnt}), \text{p} \cdot (\text{cnt} + 1)] \cdot \text{expp}; \]
\[ \text{temp1} = \text{sparse}([\text{temp1}; \text{zeros}(1, \text{cnt}), \text{base\_a} \cdot \text{expp}, \text{zeros}(1, \text{unkn} - \text{cntt} \cdot \text{length(base\_a}))]; \]
\[ \times \text{zeros}(1, \text{cnt}), \text{base\_b} \cdot \text{pexpp}, \text{zeros}(1, \text{unkn} - \text{cntt} \cdot \text{length(base\_a})); \]
\[ \text{end} \]
\% Linear equation \text{out\_m1} \cdot \text{unknown} = \text{out\_m2}
out_m1=sparse([upper_m;temp1;botto_m]);
out_m2=sparse([-1,-q(cnt_a2);zeros(unkn-2,1)]);

% solving equation

% SVD method
tolera=1e-32;
solut2=sparse(svd_equ(out_m1,tolera))*out_m2;
AA2(cnt_a2)=solut2(1);

end
A2=AA2;
end

A.4 Sub function ‘svd_equ’

function y=svd_equ(svdarray,tol)
    [U S V]=svd(full(svdarray));
    dimen=length(S(:,1));
    S_1=zeros(dimen,dimen);
    for iii=1:dimen
        switch isreal(S(iii,iii))
            case 0
                if (real(1/S(iii,iii))<tol | imag(1/S(iii,iii))<tol)
                    S_1(iii,iii)=0;
                else
                    S_1(iii,iii)=1/S(iii,iii);
                end
            otherwise
                if (1/S(iii,iii)<tol)
                    S_1(iii,iii)=0;
                else
                    S_1(iii,iii)=1/S(iii,iii);
                end
        end
    end
    y=V*S_1*(U');
B. Establishment of the database used in inverse scheme

B.1 Main function

clc;
clear all
tic;

% % ---fft---

warning('Initialising..');

fs=600000;  % sampling frequency
number_sample=6000+1;  % number of samples---odd number=> last digit=1
time_total=(number_sample-1)/fs;  % total time---start from 0->'-1'
delta_freq=1/time_total;

t=0:(number_sample-1)/2, -(number_sample-1)/2:(-1)];
freq=delta_freq;

half_p=(number_sample-1)/2;
sig_v=5;
R_coil=10;
sig_amp=sig_v/R_coil;
tao0=50/fs;  % time constant

sig1=0.5*(1-exp(-tt(1:half_p+1)/tao0)).*sign(tt(1:half_p+1));
sigv=sig_v*sign(tt(1:half_p+1));
ocur=[sig1,sig_amp-sig1(2:half_p+1)];
vc=[sigv,sig_v-sigv(2:half_p+1)];

cur=fft(o cur);

% % Some preliminary definitions and input data%
kHz=1.0e3; mm=1.0e-3; MSm=1.0e6; m0=pi*4.0e-7; inc=25.400051*mm;  % Wire turns and input current
WT=230;

% Conductivities of the top (1) and bottom (2) layers
con1_ratio=linspace(0.5,2, 21); con1=con1_ratio*26.6*MSm;

% Relative magnetic permeabilities of the top (1) and bottom (2) layers
mr1=1.0; mr2=1.0;

% lift-off
lo=0.5*mm+(mm*((10.^((1:21)-1))/10))-1;
% lift-off variation
\[
\text{lov}=0.5\text{mm};
\]
\[
\text{r}_1=(9.5\text{mm})/2; \text{r}_2=(18.4\text{mm})/2;
\]
\[
\text{z}_1=\text{lo}; \text{z}_2=z_1+8.2\text{mm};
\]
\[
\text{WTD}=(\text{WT}^*\text{cur}/((\text{r}_2-\text{r}_1)*8.2\text{mm}));
\]
\[
\text{coef}=\mu_0*\text{WTD};
\]
\[
\text{h}=20*\text{r}_2; \text{Ns}=200;
\]
\[
\text{J}_1\text{roots}=\text{besselzeros}(\text{Ns});
\]
\[
q=\text{J}_1\text{roots}/\text{h};
\]
\[
\text{r}_\text{hall}\text{ratio}=0;
\]
\[
\text{r}_\text{hall}=\text{r}_\text{hall}\text{ratio}^*\text{r}_1;
\]
\[
\text{z}_\text{hall}=\text{lo}-0.5\text{mm};
\]
\[
\text{k}=0:1:11; \text{nu}=2^*\text{k}+1;
\]
\[
\text{x}_1=q^*\text{r}_1;
\]
\[
\text{x}_2=q^*\text{r}_2;
\]
\[
\text{J}_\text{x}=1:1:\text{Ns};
\]
\[
\text{for} \text{ cntj}=1:\text{Ns}
\]
\[
\text{J}_\text{x}^*(\text{cntj})=\text{quadl}(@\text{yy},\text{x}_1(\text{cntj}),\text{x}_2(\text{cntj}),1e^-6);
\]
\[
\text{end}
\]
\[
\text{bes}=h^*q^*\text{besselj}(0,q^*\text{h});
\]
\[
\text{oros}=\text{J}_\text{x}^*/(\text{bes}^*.2);
\]
\[
\text{dz}_1=\text{zeros}(\text{length(}\text{lo}),\text{length(}\text{freq}));
\]
\[
\text{dz}_2=\text{zeros}(\text{length(}\text{lo}),\text{length(}\text{freq})); \% \text{with LO variation}
\]
\[
\text{dzifft1}=\text{zeros}(\text{length(}\text{lo}),\text{length(}\text{freq}));
\]
\[
\text{dzifft2}=\text{zeros}(\text{length(}\text{lo}),\text{length(}\text{freq})); \% \text{with LO variation}
\]
\[
\text{p}_1=\text{zeros}(\text{length(}\text{freq}),\text{Ns});
\]
\[
\text{all}_\text{ loi}=\text{zeros}(\text{length(}\text{con1}),\text{length(}\text{lo}));
\]
\[
\text{refl}=\text{zeros}(\text{length(}\text{freq}),\text{Ns});
\]
\[
\text{eqz}_=\exp(-q^*0.5^*\text{mm})-\exp(-q^*8.7^*\text{mm});
\]
\[
\text{eqz}_1=\exp((z_\text{hall}+z_1)^*(-q))-\exp((z_2+z_\text{hall})^*(-q));
\]
\[
\text{eqz}_2=\exp((z_\text{hall}+z_1+2^*\text{lo})^*(-q))-\exp((z_2+z_\text{hall}+2^*\text{lo})^*(-q));
\]
disp('Starting Loop...');

for cnt_cond=1:length(con1);

  for cnt_freq=1:length(freq)
    p1=sqrt(q.*q+i*2*pi*freq(cnt_freq)*m0*mr1*con1(cnt_cond));
    refl(cnt_freq,:)=(q-(p1/mr1))./(q+(p1/mr1));
  end

  for cntt=1:length(lo)
    for cnt_f=1:length(freq)

      % reflection

      % % % change hall sensor position % % %
      orosf1 = besselj(0,q*r_hall).*oros.*(eqz_+eqz1(cntt,:).*refl(cnt_f,:));
      orosf2 = besselj(0,q*r_hall).*oros.*(eqz_+eqz2(cntt,:).*refl(cnt_f,:));  % with LO variation
      dz1(cntt,cnt_f)=coef(cnt_f)*sum(orosf1);
      dz2(cntt,cnt_f)=coef(cnt_f)*sum(orosf2);

    end

    % IFFT
    dzifft1(cntt,:)=ifft(dz1(cntt,:));
    dzifft2(cntt,:)=ifft(dz2(cntt,:));

  end

  % % LOI computation

  % find LOI
  for cnt_lo=1:length(lo)

    all_loi(cnt_cond,cnt_lo)=findloi(tt(1:1000),dphidt(tt(1:1000),dzifft1(cnt_lo,1:1000)),dphidt(tt(1:1000),dzifft2(cnt_lo,1:1000)));

  end

  disp(['finished step: ', num2str(100*(cnt_cond/length(con1))),' %']);

end

B.2 Sub function ‘dphidt’

function yy=dphidt(t, x)
pp=spline(t, x);
dpp=fnder(pp);
yy=ppval(dpp,t);

B.3 Sub function ‘findloi’

function yy=findloi(t,a,b)

% Find the intersection point between 2 signals 15/12/2007
% Usage: findloi(t, a, b)
% t --- time, its length is equal to that of each signal
% a ---Signal 1
% b ---Signal 2
% Warning: The built-in subfunction 'fnzeros' should be in Matlab directory.

pp=spline(t,a-b);
temp=fnzeros(pp);
yy=temp(1,1);
List of Publications

Journal papers


Conference papers


