The Wave Function and Quantum Reality

Shan Gao

Unit for History and Philosophy of Science & Centre for Time, SOPHI
University of Sydney, Sydney, NSW 2006, Australia

Abstract. We investigate the meaning of the wave function by analyzing the mass and charge density distributions of a quantum system. According to protective measurement, a charged quantum system has effective mass and charge density distributing in space, proportional to the square of the absolute value of its wave function. In a realistic interpretation, the wave function of a quantum system can be taken as a description of either a physical field or the ergodic motion of a particle. The essential difference between a field and the ergodic motion of a particle lies in the property of simultaneity; a field exists throughout space simultaneously, whereas the ergodic motion of a particle exists throughout space in a time-divided way. If the wave function is a physical field, then the mass and charge density will be distributed in space simultaneously for a charged quantum system, and thus there will exist gravitational and electrostatic self-interactions of its wave function. This not only violates the superposition principle of quantum mechanics but also contradicts experimental observations. Thus the wave function cannot be a description of a physical field but be a description of the ergodic motion of a particle. For the later there is only a localized particle with mass and charge at every instant, and thus there will not exist any self-interaction for the wave function. It is further argued that the classical ergodic models, which assume continuous motion of particles, cannot be consistent with quantum mechanics. Based on the negative result, we suggest that the wave function is a description of the quantum motion of particles, which is random and discontinuous in nature. On this interpretation, the square of the absolute value of the wave function not only gives the probability of the particle being found in certain locations, but also gives the probability of the particle being there. The suggested new interpretation of the wave function provides a natural realistic alternative to the orthodox interpretation, and it also implies that the de Broglie-Bohm theory and many-worlds interpretation are wrong and the dynamical collapse theories are in the right direction by admitting wavefunction collapse.

Keywords: wave function; charge density; protective measurement; ergodic motion of particles
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1. INTRODUCTION

The wave function is the most fundamental concept of quantum mechanics. According to the standard probability interpretation, the wave function is a probability amplitude, and the square of its absolute value represents the probability density for a particle to be measured in certain locations. However, this interpretation is unsatisfying when applying to a fundamental theory because of resorting to measurement. In view of the problem, some alternative realistic interpretations of the wave function have been proposed and widely studied [1-4]. There are in general two ways to interpret the wave function of a single quantum system in a realistic interpretation[1]. One view is to take the wave function as a physical entity existing throughout space simultaneously such as a field [1,2,4]. The other view is to take the wave function as a description of some kind of ergodic motion of a particle [3]. In this paper[2], we will argue that these two interpretations of the wave function can in fact be tested by analyzing the mass and charge density distributions of a quantum system, and the former has already been excluded by experimental observations. Moreover, a further analysis can also determine which kind of ergodic motion of particles the wave function describes. The motion turns out to be random and discontinuous in nature.

2. PROTECTIVE MEASUREMENT AND CHARGE DENSITY

The mass and charge of a charged classical system always localize in a definite position in space at each moment. For a charged quantum system, how do its mass and charge distribute in space then? Although this question seems meaningless according to the probability interpretation of the wave function, it should have a physical meaning in a realistic interpretation of the wave function. We can measure the total charge of a quantum system by electromagnetic interaction and find them in some region of space after all. It can be reasonably guessed that a quantum system has mass and charge density distributing in space, proportional to the square of the absolute value of its wave function [5]. This is also a consequence of protective measurement; the mass and charge density can be measured by protective measurement as expectation values of certain variables for a single quantum system [6,7].

Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. A protective measurement of an observable A_n, which is a normalized projection operator on small regions V_n having volume v_n, will yield the following result [7]:

$$\langle A_n \rangle = \frac{1}{v_n}\int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2 \qquad (1)$$

It is the average of the density $|\psi(x)|^2$ over the small region V_n. When $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can find the whole density distribution $|\psi(x)|^2$. For a charged system with charge *Q*, the density $|\psi(x)|^2$ times the charge yields the effective charge density $Q|\psi(x)|^2$. In particular, an appropriate adiabatic measurement of the Gauss flux out of a certain region will yield the value of the total charge inside this region, namely the integral of the effective charge density $Q|\psi(x)|^2$ over this region [7]. Similarly, we can measure the effective mass density of the system in principle by an appropriate adiabatic measurement of the flux of its gravitational field. Therefore, protective measurement shows that the mass and charge of a single quantum system described by the wave function $\psi(x)$ is distributed throughout space with effective mass density $m|\psi(x)|^2$ and effective charge density $Q|\psi(x)|^2$ respectively.

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[1] For the sake of simplicity, we will mainly discuss the wave function of a single quantum system in this paper. The conclusion can be readily extended to many-body system, which wave function is defined in configuration space.

[2] An enlarged version of this paper is available online at PhilPapers [5].

3. WHY THE WAVE FUNCTION IS NOT A PHYSICAL FIELD

Although protective measurement strongly suggests a realistic interpretation of the wave function, it does not directly tell us what the wave function is. The wave function may describe a physical filed or some kind of ergodic motion of a particle. Correspondingly, the mass and charge density may result from a physical field or the ergodic motion of a particle. These two explanations are essentially different in that a field exists throughout space simultaneously, whereas the ergodic motion of a particle exists throughout space in a time-divided way.

If the wave function of a quantum system is a physical field, then its mass and charge density will simultaneously distribute in space. As a result, different spatial parts of the wave function will have gravitational and electrostatic interactions, as these parts have mass and charge simultaneously. Then the Schrödinger equation for a free quantum system with mass *m* and charge *Q* will be

$$i\hbar\frac{\partial\psi(x,t)}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial^2 x}+(kQ^2-Gm^2)\int\frac{|\psi(x',t)|^2}{|x-x'|}d^3x'\,\psi(x,t) \qquad (2)$$

where *k* is the Coulomb constant, and G is Newton's gravitational constant.

It has been shown that the measure of the potential strength of a gravitational self-interaction is $\varepsilon^2=(4Gm^2/\hbar c)^2$ for a free system with mass *m* [8]. This quantity represents the strength of the influence of self-interaction on the normal evolution of the wave function; when $\varepsilon^2\approx 1$ the influence will be significant. Similarly, for a free charged system with charge *Q*, the measure of the potential strength of the electrostatic self-interaction is $\varepsilon^2=(4kQ^2/\hbar c)^2$. For example, the potential strength of the electrostatic self-interaction is $\varepsilon^2=(4ke^2/\hbar c)^2\approx 1\times 10^{-3}$ for a free electron. This indicates that the electrostatic self-interaction will have significant influence on the evolution of its wave function. If such an interaction indeed exists, it should have been detected by precise experiments. As another example, consider the electron in the hydrogen atom. Since the potential of its electrostatic self-interaction is of the same order as the Coulomb potential produced by the nucleus, the energy levels of hydrogen atoms will be significantly different from those predicted by quantum mechanics and measured by experiments. Therefore, the electrostatic self-interaction cannot exist. Since the field explanation of the wave function entails the existence of such electrostatic self-interactions, it cannot be right, i.e. the wave function cannot be a description of a physical field.

4. TOWARDS QUANTUM MOTION OF PARTICLES

The failure of the field interpretation leads us to the second view that takes the wave function as a description of some sort of ergodic motion of particles. On this view, the effective mass and charge density are formed by time average of the motion of a charged particle, and they distribute in different locations at different moments. Thus there will not exist any self-interaction for the wave function. In fact, if the mass and charge density does not exist in different regions simultaneously as the field interpretation holds, they can only exist throughout space in a time-divided way. As a result, the wave function must be a description of the ergodic motion of particles.

It can be further argued that the classical ergodic models that assume continuous motion of particles cannot be consistent with quantum mechanics [5,7][3]. These models are plagued by the problems of infinite velocity, accelerating radiation and the existence of a finite time scale etc [5,7]. In view of this negative result, it has been suggested that another different kind of motion – random discontinuous motion can naturally generate the effective mass and charge density measurable by protective measurement, and what the wave function describes is probably such quantum motion of particles, which is essentially discontinuous and random [11,12].

If the motion of a particle is not continuous but discontinuous and random, then the particle can readily move throughout all possible regions where the wave function spreads during an arbitrarily short time interval near a given instant. This will solve the problems of classical ergodic models [5]. In fact, by assuming the wave function is a (complete) description for the actual motion of particles, we can reach the random discontinuous motion in a more direct way. If the wave function $\psi(x,t)$ is a description of the state of motion for a single particle, then the quantity $|\psi(x,t)|^2 dx$ will not only give the probability of the particle being found in an infinitesimal space interval dx near position x at instant t (as in standard quantum mechanics), but also give the objective probability of the particle being there. This accords with the reasonable expectation that the probability distribution of the measurement outcomes of a property is the same as the actual distribution of the property in the measured state. Obviously, this kind of motion is essentially random and discontinuous.

The strict mathematical description of random discontinuous motion (RDM henceforth) can be obtained by using the measure theory. It has been shown that the position measure density $\rho(x,t)$ and the position measure flux density $j(x,t)$ provide a complete description for the RDM of a single particle [12]. By assuming that the nonrelativistic evolution equation of RDM is the Schrödinger equation, the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$, and thus it also provides a complete description of the RDM of a single particle.

The new interpretation of the wave function in terms of RDM of particles provides a natural realistic alternative to the orthodox view. On this interpretation, the square of the absolute value of the wave function not only gives the probability of a particle *being found* in certain locations, but also gives the objective probability of the particle *being* there. Certainly, the transition process from "being" to "being found", which is closely related to the notorious quantum measurement problem, also needs to be explained. This issue will be discussed in the next section.

5. FURTHER DISCUSSIONS

If the wave function is really a description of quantum motion of particles, which is random and discontinuous in nature, then the main realistic interpretations of quantum mechanics will be either rejected or revised.

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[3] It has been pointed out that the classical stochastic interpretations (e.g. [3]) are inconsistent with quantum mechanics [9,10].

First, the de Broglie-Bohm theory will be wrong. The theory takes the wave function as a physical field (i.e. Ψ-field) and further adds the non-ergodic motion of Bohmian particles to interpret quantum mechanics. This is obviously inconsistent with the above result. As argued previously, taking the wave function as a field will lead to the existence of electrostatic self-interaction that contradicts both quantum mechanics and experimental observations. Moreover, inasmuch as the wave function has charge density distribution in space for a charged quantum system, there will also exist an electromagnetic interaction between it and the Bohmian particles. This is also inconsistent with quantum mechanics[4].

Next, the ontology of the many-worlds interpretation and dynamical collapse theories needs to be revised from field to particle. Besides, it can be further argued that there is only one world and quantum mechanics is also a one-world theory. The key point is that quantum superposition exists in a form of time division by means of the RDM of particles, and there is only one observer (as well as one quantum system and one measuring device) all along in a continuous time flow during quantum evolution [5]. Thus the many-worlds interpretation will be wrong too. Moreover, there must exist an objective process of wavefunction collapse, which is responsible for the transition from microscopic uncertainty to macroscopic (approximate) certainty. Therefore, the dynamical collapse theories will be in the right direction.

It has been argued that the discreteness of spacetime may inevitably result in the collapse of the wave function, and the compete evolution law of RDM in discrete spacetime will naturally include the dynamical collapse of the wave function. In particular, the motion of particles just provides the random source to collapse the wave function [11,12]. This may be a promising start. But more study is still needed before we can solve the quantum measurement problem (e.g. preferred basis problem) and finally understand the meaning of quantum theory.

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[4] One may want to deprive the Ψ-field of mass and charge density to eliminate the electrostatic self-interaction. But, on the one hand, the theory will break its physical connection with quantum mechanics, as the wave function in quantum mechanics has mass and charge density, and on the other hand, since protective measurement can measure the mass and charge density for a single quantum system, the theory will be unable to explain the measurement results either. Although de Broglie-Bohm theory can still exist in this way as a mathematical tool for experimental predictions, it obviously departs from the initial expectations of de Broglie and Bohm, and as we think, it already fails as a physical theory because of losing its explanation ability.