Evolutionary game and dynamics in our networking life: More than cooperation

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Outline

- Evolutionary game and networks
  
  **Cooperation: motivation, rules, and models**

- From cooperation to *costly punishment*
- Cooperation and *opinion formation and dynamics*
- More than Cooperation: *behavioral diversity*

- Extension and outlook
Acknowledged Co-contributors

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Cooperation is ubiquitous in nature


COMMON VAMPIRE BATS frequently engage in acts of mutual help. A bat that feeds successfully on blood from horses or cattle will share its nourishment with an unfed companion by regurgitating a portion of its stomach contents.
Cooperation: the basis of human civilization

Robert Boyd and Sarah Mathew, *A Narrow Road to Cooperation*, SCIENCE, 2007
**Prisoner’s dilemma (PD) of two criminals**

**Cooperator:** obey the alliance with another criminal to deny the accusation  
**Defector:** defect the alliance and admit the accusation

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(deny)</td>
<td>(-2,-2)</td>
<td>(-5,-1)</td>
</tr>
<tr>
<td>D(admit)</td>
<td>(-1,-5)</td>
<td>(-3,-3)</td>
</tr>
</tbody>
</table>

If you opponent plays C: you’d better play D.  
If you opponent plays D: you’d better play D.  
**But,**  
CC is better than DD

**Dilemma:**  
Despite mutual cooperation being the best, individual tends to **DD**.
Defectors can diffuse in well-mixed populations

Prisoner's dilemma → cooperation never survives evolution

Cooperation is ubiquitous in nature! What is wrong?

Fig. 1. Without any mechanism for the evolution of cooperation, natural selection favors defectors. In a mixed population, defectors, D, have a higher payoff (= fitness) than cooperators, C. Therefore, natural selection continuously reduces the abundance, i, of cooperators until they are extinct. The average fitness of the population also declines under natural selection. The total population size is given by N. If there are i cooperators and N − i defectors, then the fitness of cooperators and defectors, respectively, is given by $f_i = [b(i - 1)/(N - 1)] - c$ and $f_D = bi/(N - 1)$. The average fitness of the population is given by $\bar{f} = (b - c)i/N$.

Natural cooperation


“The two fundamental principles of evolution are mutation and natural selection. But evolution is constructive because of cooperation. New levels of organization evolve when the competing units on the lower level begin to cooperate. …

Thus, we might add “natural cooperation” as a third fundamental principle of evolution beside mutation and natural selection.”
Some rules for evolutionary cooperation


- **Kin selection**: relative
- **Direct reciprocity**: unrelated individuals
- **Indirect reciprocity**: reputation
- **Group selection**
- **Network reciprocity**
Spatial evolutionary game

M. Nowak and R. May, Nature 1992

- **Players:** locate on a square lattice, randomly choose C or D at the initial time.
- **Play PD game with neighbors on the square lattice, and learn its best neighbor’s strategy**
Evolutionary games & spatial chaos

Temporal evolution beginning with a D in the middle

Fig. 3: Spatial games can generate an evolutionary kaleidoscope. This simulation started with a single D at the centre of a 59 x 59 square lattice world of C with fixed boundary conditions. Again L8 = δ = 2. This generates an (almost) infinite sequence of different patterns. The initial symmetry is always maintained, because the rules of the game are symmetrical. The frequency of K oscillates chaotically around a static average of 12 log 2 - 6 (of course). a: Generation t = 30; b: t = 217; c: t = 219; d: t = 221.
Snowdrift game (SG)

**Cooperator:** help others at a cost to themselves.
**Defector:** receive the benefits without providing help.

\[
\begin{array}{cccc}
\text{player 1} & & \text{player 2} \\
\begin{cases} 
C \\
D 
\end{cases} & \begin{cases} 
\frac{b-c}{2}, & b-c \\
\frac{b}{c}, & 0 
\end{cases}
\end{array}
\Rightarrow
\begin{array}{cccc}
1, & 1 - \frac{c}{2b-c} \\
1 + \frac{c}{2b-c}, & 0 
\end{array}
\Rightarrow
\begin{array}{cccc}
1, & 1 - r \\
1 + r, & 0 
\end{array}
\]

\[r = \frac{c}{2b-c} \text{: cost to benefit of mutual cooperation}\]

**C and D coexist: C\to 1-r \to D**

The equilibrium frequency of cooperators in SG is 1-r
Hauert, C., Doebeli, M. Spatial structure often inhibits the evolution of cooperation in the snowdrift game. NATURE, 2004.

(a) PD: larger, compact cluster  (b) SG: cross-like structure
How is the role of ‘small-world’, ‘scale-free’ and etc. in our networking life?
Evolutionary games on complex networks

- Cooperator frequency $f_c$

Game Model

- Selection rule
  - Best take over
  - Random
  - Preferential
  - ... i.e. PD, SG, UG, PGG

- Replacement rule
  - Replicator dynamics:
    \[ W(x<-y) = f(P_y - P_x) \]
  - Glauber dynamics:
    \[ W(x<-y) = (1 + \exp(x-y/T))^{-1} \]
  - Win stay, lost shift
  - Memory ...

Evolutionary Rule

Population Structure

- Lattice, RG, SW, SF ...
- $<k>$, gama, $r_k$, ...
The Game of Life and How to Play It

Florence Scovel Shinn (1925), 1st edition

“It asserts that life is not a battle but a game of giving and receiving, and that whatever we send out into the world will eventually be returned to us.”
More stories......

from more than Cooperation (frequency)
Other mechanism to maintain or enhance cooperation?

How about punishing defectors?


Cooperator: helps others at a cost to themselves.
Defector: receives the benefits without providing help.

Punisher: pays a cost for other (defecting) players to incur their (more) costs (costly punishment)
Winners don’t punish


- Extend the repeated PD game from C/D to C/D/P
- Strategy to defectors (104 persons involved experiments):
  Winner uses **Tit-for-Tat** (D for D)
  Loser uses **Punishment** (P for D)
Ultimatum Game (UG) & Costly Punishment

- **Proponent**: offer a division \((p, 1-p)\)
- **Respondent**: accept (get the \(1-p\)) or reject the division
- If the division is rejected, both two get nothing.

The more selfish a proponent is, the more \(p\) he keeps for himself, and the less he offers for the respondent.

A respondent rejects the division => costly punishment

• A – proponent, B – respondent;
• B – proponent, A – respondent;

• Assumption: Individuals are homogeneous with the same selfishness degree $\bar{p}$.

• Two rounds of peer-to-peer UG playing between a pair of two connected players.

$$M = \begin{pmatrix} 1 & 1 - \bar{p} \\ \bar{p} & 0 \end{pmatrix}$$
The role of empathy

• players (as proponents) make offers which are acceptable for themselves.
  (己所不欲,勿施于人)


• Extension the role of empathy: adjusts his accepting probability by comparing his payoff ratio with the average level of all his connected neighbors.

**consensus-protocol-type rule**

- If one player rejects offers with a larger (smaller) probability compared with his neighbors, he is hence inclined to increase (decrease) to accept (reject) more

- **consensus of the accepting probability** $r_i$

\[ \dot{r}_i = \frac{1}{k_i} \sum_{j \in N_i} \tilde{u}_j - \tilde{u}_i \]

The emergence of costly punishers


The **Sufficient and Necessary** condition:

\[ \bar{p} > p_c = \frac{1}{\tilde{\lambda}_n} \]

A selfish proponent keep the part for himself in a division

\[ 0 = \tilde{\lambda}_1 \leq \tilde{\lambda}_2 \leq \cdots \leq \tilde{\lambda}_n \leq 2 \]

eigenvalues of the normalized Laplacian

Population **selfish degree** &

Population **connectivity patterns**
The more selfish a division from the proponent ($0.5<p<1$), the more possible to punish from the respondent.

Winners don’t punish (not very selfish opponent)
Winners DO punish (too selfish opponent)


SCN,
N=9842
E=37786
Cooperation & opinion formation

Battle-of-Sexes Game

<table>
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<tr>
<th></th>
<th>opera</th>
<th>boxing</th>
</tr>
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<tbody>
<tr>
<td>opera</td>
<td>$(2x, x)$</td>
<td>$(0, 0)$</td>
</tr>
<tr>
<td>boxing</td>
<td>$(0, 0)$</td>
<td>$(x, 2x)$</td>
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</tbody>
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When both sides cooperate to form the same opinion, then they both can obtain some gains. Otherwise, none.
Opinion formation over complex networked players

When extended from a couple to a population of players, how is the role of connectivity patterns of population between the opinion formation and gaming cooperation?

L Cao, X. Li (2008), Physical Review E, 77:016108
Strategy updating:
Birth-death and Death-birth processes

- Convincing process, natural selection
Opinion dynamics & network connectivity

Coexisting or dominated opinions implied by strategies among different network communities?

A Network with community structure

L. Cao, X. Li (2008), Physical Review E, 77:016108
Network normalization

Graph $G$ with different classes of opinions $\rightarrow$
Class-Adjacency-Strength matrix $W$
Main contributions

• The stability analysis on the coexistence of mixed strategies among multi-class individuals is given.

• Individuals of a community or modular structured network are prone to form coexisting opinions, and the coexistence of mixed evolutionary strategies implies the modularity of networks.

L. Cao, X. Li (2008), Physical Review E, 77:016108
Opinion dominance and coexistence phases

Network modularity

\[ M = \frac{1}{2} - \frac{p_1 + p_2}{2} \]

Vs

Opinion dominance

L. Cao, X. Li (2008), Physical Review E, 77:016108
“The ultimate understanding of the overall behavior of a complex network must, however, account for its architecture as well as the nature of dynamical processes taking place on such a network.”

Behaviors of decisions and choices

Operating systems:
Windows, Mac, Linux...

Party: Democracy, Republican

Laptop: Dell, Hp, Sony, Lenovo...

Fashion: classic, hip-hop, cool...
Behavior networks

Related behaviors can be mapped into a weighted network:

A vertex represents a behavior.

An edge means two behaviors can mutually convert.

A behavior $b_i$ converts to a behavior $b_j$ with reward $a_{ij} = a_{ji}$ ($a_{ii} = 1$, $a_{ij} < 1$)

Fraction of behavior: $x_i \rightarrow \sum_i x_i = 1$

$A = [a_{ij}]$

A behavior network

Model Formation

**Behavior networks**

A behavior gains the utility (fitness) in proportion to total reward from all the people.

\[ f_i = \sum_j a_{ij} x_j \]

fitness of behavior

\[ q_{ij} = \mu(a_{ij} / \sum_j a_{ij}) \]

Behaviors mutually conversion rate

\[ q_{ii} = 1 - \mu(a_{ii} / \sum_j a_{ij}) \]

mutation rate (tunable parameter)
Model Formation (cont.)

**Behavior networks**

\[ \dot{x}_i = \sum_{j=1}^{n} f_j x_j q_{ji} - \phi x_i \]

System evolves as replicator-mutator dynamics

\[ \phi = \sum_j f_j x_j \] average fitness

promise \[ \sum_i x_i = 1 \]

In steady state, we focus on the behavioral diversity.

Behavioral diversity

\[ n_e(x) = \frac{1}{\sum_i x_i^2} \]

- **consensus** \( n_e = 1 \)
- **cohesion** \( 1 < n_e \ll n \)
- **collapse** \( 1 \ll n_e < n \)
- **complete collapse** \( n_e = n \)
Y. Yang, X. Li, Z.H. Rong (2010), *Assortative degree-mixing patterns inhibit behavioral diversity of a scale-free structured population in high mutation situations*, *EPL*, 89, 18006
General summary

- **Evolutionary Game is not limited to Cooperation**

- Networking enriches the evolutionary dynamics of game-players

- We all are learning from others, and also adaptively improve ourselves. (More complexity)
One extension

• Snowdrift Game and (minimal) Vertex Cover \( \text{(Karp’s 21 NP-complete problems)} \)

-----left for next time. 😊

• Wiki: A vertex cover of a graph \( G \) is a set \( C \) of vertices such that each edge of \( G \) is incident to at least one vertex in \( C \). The set \( C \) is said to cover the edges of \( G \).
This is not the End!
It is not even the beginning of the end!

—— Sir W. L. S. Churchill

We CAN do our best to continue it!

CAN: Adaptive Networks and Control Laboratory

自适应网络与控制研究室·复旦大学
自适应网络与控制研究室招收博士后

复旦大学电子工程系拥有电路与系统国家重点二级学科和生物医学工程上海市重点学科，中国工程院院士1名，是“电子科学与技术”（国家重点一级学科）、“生物医学工程”两个博士后流动站核心组成部分。

电子工程系主任李翔教授主持的自适应网络与控制研究室长期致力于在复杂网络与复杂系统的理论及其应用领域开展科学创新研究工作，计划2010-2011年招收全职在站博士后研究人员2名。
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Thank you!