Agricultural rotavator power requirement optimization using multi-objective probability parameter optimization

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Abstract: The rotavator is a tillage tool capable of creating the desired soil tilth quality with significantly fewer tillage passes. In this paper, a power consumption analytical model of a typical L-shaped blade for a small rotavator, which is suitable for soil cultivation of hills and mountainous areas of southern China, was deduced. Then a power requirement optimizing method with a multi-objective parameter design for rotavator soil tilling such as forward travel speed, power shaft rotation velocity and soil pulverizing effect, was developed, which is a probability parameter optimization method based on index atlases, written in the MATLAB program. A rotavator power requirement optimization method was developed using a multi-objective parameter design to optimize forward travel speed, power shaft rotation velocity and soil pulverizing effect. This probability parameter optimization method is based on index atlases written in the MATLAB program. In order to reduce peak power required and to guarantee the soil pulverizing effect, a case study of optimizing forward travel speed and power shaft rotation velocity was tested and verified. Moreover, other working speeds optimization cases for tilling various soils with a small rotavator were also carried out. The results show that the proposed models and methods are valid and available.

Keywords: rotavator, multi-objective optimization, power, working speed


1 Introduction

The rotavator is a tillage tool primarily comprising of L-shaped blades mounted on flanges that are affixed to a shaft that is driven by the tractor power-take-off (PTO) shaft. In comparison to passive tools, the rotavator has a superior soil mixing and pulverization capability. During rotavator tillage operations various factors affect its energy requirements. These factors can include soil conditions, operational conditions and rotavator configuration.

Discussing the influence of mechanical cultivation on the physical soil characteristics and the changing law of crop growth has been of widespread concern, and still receives attention from soil and agriculture science workers in order to provide a theoretical basis for cultivation (Kosmas et al., 2001; Hammad and Dawelbeit, 2001; Cameira, Fernando and Pereira, 2002). For pulverizing rate, different pulverizing evaluation indices of various soils are mainly obtained by means of experiments, which did not form a very complete theoretical system and had limited guiding significance to actual operation. Thakur and Godwin found out that the greatest force of rotary tool occurs when the cutting angle is $\pi/18$–$\pi/12$ rad under the condition of quasi-static contact and one-cycle rotating process. This discovery provides an important mechanical basis for the design of the rotary tool.

In the aspects of saving power consumption and improving energy saving of agricultural machinery during soil cultivation, many scholars made a lot of research and practice on the influences of turning direction of rotary tool to cultivation and energy saving effect (Manian and Kathirvel, 2001; Prasad, 1996). In attempt to quantify
the energy requirement of tillage tools, a number of models have been developed that relate the input and output parameters. Such models predict the forces acting on soil tillage tools in relation to tool geometry, physical soil properties and the nature of soil disturbance ahead of the tool (Godwin and O’Dogherty, 2006; Gill and Vanden Berg, 1968). Though numerous attempts have been made to quantify the performance of rotavators, little is known about how the rotavator design parameters and soil conditions influence the energy required and quality of work due to the experience or experimental nature of the studies on performance (Marenya, du Plesis and Musonda, 2003).

In order to understand the influence of tool design parameters and the influence of soil conditions on performance, Gill and Vanden Berg (1968) emphasized that mathematical description of the tillage process can be accomplished only when all the elements of the tillage process are expressed quantitatively. They proposed a hypothetical model to illustrate the factors involved in influencing the desired quality of operation and resulting forces from the design point of view. In our study, the rotavator needs to be equipped with different capacity powers and blade configurations according to various field environments or machine working conditions, such as physical soil character, steering space, tillage speed and tillage efficiency. Moreover, a multi-objects optimizing model, which considered the initial soil conditions, shape and the manner of movement of the tool as input factors, and blade tilling power and the final soil condition as output parameters, was proposed as a basis for configuring the power capacity and blade configuration of an agricultural rotavator fitted with L-shaped blades.

2 Kinetic parameters of a rotavator fitted with typical L-shaped blades

A hypothetical soil-tool interaction model of the down-cut rotavator fitted with an L-Shaped blade is shown in Figure 1. It was assumed that the face CDF separated from the uncut soil body breaks away in a vertical manner, and the rotavator slices the soil into a soil wedge with the forward velocity $V_f$, rotational speed $V_r$, and soil wedge width $w$ that corresponds to the span of the blade. The set tillage depth determines the angle at which the cutting edge of the blade comes into contact with the soil surface and the angle at which the blade stops the soil cutting process, and a kinematic parameter $\lambda$ that is introduced (Hendrick and Gill, 1971a, b, c). This dimensionless ratio is expressed as:

$$\lambda = \frac{V_r}{V_f} = \frac{R \omega}{V_f}$$

(1)

Where: $R$ rotor radius, m; $\omega$ angular velocity of the power shaft, rad/s; $V_f$ forward travel velocity, m/s; $V_r$ peripheral velocity of the blades, m/s.

From Figure 1, for a down-cut direction of rotation at a given kinematic parameter $\lambda$, the length of cut, $L_{ct}$ is dependent on the set depth of tillage. $w$ is width of the blade, $d$ is depth of tillage, and $L_b$ is bite length of soil-tool interaction.

In rotavators, the forced rotation of the power shaft, with working tools fixed to it, participates in two motions, namely, (1) rotary motion around its axis with velocity $V_r$ and forward travel speed $V_f$. For deriving the equation of motion, a stationary system of co-ordinates is considered, which is shown in Figure 2.

The point $A_1$ corresponds with the tip of the tillage blade. The distance from the rotational axis to the point of interest points $A_2$ move a distance equal to $V_f t$, and take positions $A_3$ The co-ordinates of these points are expressed (Hendrick and Gill, 1971; Bernacki, Haman and Kanafojski, 1972) as:
Where: $\alpha$ is the angle of rotation of the tillage blade with respect to its initial position, rad; and $t$ is the time of rotation of the power shaft through angle $\alpha$, s.

Figure 2  Diagram for the determination of the equation of motion  
(Sineokov and Panov, 1978)

Substituting the value of $\lambda$ into equation (2) gives the trajectory of motion of the tillage blades as:

\[
\begin{align*}
x &= R(\alpha / \lambda \pm \cos \alpha) \\
y &= R(1 - \sin \alpha)
\end{align*}
\]

(3)

Seen from equation (3), the variation of the trochoidal motion (Figure 2) is only affected by $\lambda$, characterizing the working regime of rotavators. Then the tillage speed or absolute velocity of the tillage blades is obtained by differentiating equation (3) with respect to time. Therefore,

\[
\begin{align*}
V_x &= \frac{dx}{dt} = V_x \mp V_y \sin \omega t \\
V_y &= \frac{dy}{dt} = -V_x \cos \omega t
\end{align*}
\]

(4)

And the modulus of the absolute velocity, $V_o$, of the tillage blade tip (Sineokov and Panov, 1978) is given by:

\[V_o = \sqrt{V_x^2 + V_y^2} = V_f \sqrt{V_f^2 + 2V_f V_y \sin \omega t + V_y^2}
\]

(5)

Inserting $\lambda$ and $\alpha = \omega t$ into equation (5) gives,

\[V_o = V_f \sqrt{\lambda^2 + 2\lambda \sin \alpha + 1}
\]

(6)

Bite length, $L_b$, is one of the principal technological parameters of the rotavator (Chamen, Cope and Patterson, 1979) and determines how finely chopped processed soil will be (Sineokov and Panov, 1978). The value of $L_b$ is given by:

\[L_b = V_f t_b
\]

(7)

Where: $t_b$ is the time during which the blades rotate through an angle equal to the angle between the adjacent blades on the same side of a flange.

If the number of blades in one plane of the power shaft flange is $z$, the angle between adjacent blades is $2\pi/z$ radians. In such a case, the time $t_b$ is given by $t_b = 2\pi/z \omega$ and the bite length $L_b$ is expressed as:

\[L_b = 2\pi V_f / (z \omega)
\]

or

\[L_b = 2\pi R / (z \omega)
\]

(8)

3  Modeling tillage blade energy requirements

In processing the soil using the L-shaped blade, two of its distinct parts come into contact with the soil, viz., the vertical section (called the stem or leg), and the horizontal section (called the span). As seen in Figure 1, the stem requires torque for cutting and separating the cut soil slice along face ABE, and for overcoming the soil-metal friction on the blackside of the leg in contact with the uncut soil mass. The span requires torque for overcoming soil shear strength, soil-metal friction (on the inside), cutting by the leading edge, and the acceleration and throwing of the cut soil slice.

In order to predict a rotavator’s torque requirements, the next step was to identify the soil-tool interaction forces on the different planes that would enable the development of a soil-tool interaction forces model. The 3-D resistance model predicting soil resistance, Perumpal-Desai-Grisso model (Perumpal, Grisso and Desai, 1983) based on the limit equilibrium analysis was used in this paper. Based on soil wedge failure, all the forces acting on the different surfaces are identified as shown in Figure 3, and these forces contribute to the total force due to the wedge, designated as $P_s$, which act at an angle $\delta$, normal to the surface of the span. The description of all identified force components, per wedge surface, is as follows:

- Rectangular surface abed: (1) Adhesion force, $F_{ad}$ due to adhesion between the span and the soil; (2) Soil-metal force on the interface between the soil and the span of the blade; and (3) The force exerted on the span by the tool in the instantaneous direction of movement, $P_s$.

- Triangular surface abc: (1) Equal and opposite reaction normal forces, $R$; (2) Cohesion force, $CF_2$ due to soil cohesion between the soil particles; and (3) Friction force, $SF_2$ between soil particles, i.e., soil-soil friction.
• Rectangular rupture surface bcfe: (1) The normal acting force, \( Q \); (2) Cohesion force, \( CF_1 \) due to cohesion between soil particles; and (3) Friction force, \( SF_1 \) due to soil-soil friction between the soil particles on this plane.

• Triangular surface def: (1) Equal and opposite reaction normal forces, \( R \); (2) Cohesion force, \( CF_2 \) due to soil cohesion between the soil particles; and (3) Friction force, \( SF_3 \) due to soil-metal friction between the leg and the soil.

• Other additional forces identified due to the failed soil wedge were: (1) Acceleration force \( F_{ac} \), a body force resisting the acceleration of the wedge; (2) Gravitational force, \( W \), due to the weight of the wedge; and (3) Surcharge force, \( F_q \), due to the heaving of the soil in front of the span.

• The angle used in Figure 3 is defined below: \( \beta \): angle that the tool makes with the horizontal during an instantaneous time moment, this is also called the rake angle; \( \rho \): angle that the rupture surface makes with the horizontal; \( \delta \): soil-metal friction angle; and \( \phi \): soil internal friction angle.

![Figure 3](image)

Figure 3  Breakdown of the soil-tool interaction forces on an idealized soil wedge for the span at an instantaneous time moment for a down-cut rotavator tillage operation

The total soil resistance force, \( P_s \), was obtained using the approach of Perumpral, Grisso and Desai (1983), and Swick and Perumpral (1988). The resultant equation for the total resistance force obtained was of the following form.

\[
P_s = -F_{ac} \cos(\beta + \phi + \rho) + (W + F_q) \sin(\phi + \rho) + (CF_1 + CF_2 + SF_2 + SF_3) \cos \phi / \sin(\beta + \phi + \rho + \delta)
\]

Where:

\[
\begin{align*}
F_{ac} &= C_a \times A_1 = C_a \times w \times L \\
W &= \gamma \times w \times A_2 \\
CF_1 &= C_c \times A_1 \\
SF_1 &= C_s \times A_1 \\
SF_2 &= R' \times \tan \phi \\
SF_3 &= A_2 \times C_s \\
CF_2 &= R' \times \tan \phi \\
R &= \gamma \times (1 - \sin \phi) \times d \times A_2
\end{align*}
\]

Summation of the component forces in the X-axis and Y-axis directions that enabled the derivation of the total soil separation resistance is as follows:

\[
\begin{align*}
\sum P_x &= P_s \cos(\beta + \delta) \\
\sum P_y &= P_s \sin(\beta + \delta)
\end{align*}
\]

The approach of Perumpral, Grisso and Desai (1983) can be used to solve equation (11) using a computer program, written in MATLAB. This program calculates the force \( P_s \) for different values of the rupture angle as the blades process soil. Then the rotavator tillage power can be obtained and calculated by the following expressions:

\[
M = \sum P_s \times V_x + \sum P_s \times V_y
\]

4  Multi-object optimization rotavator model

4.1 Probability parameter optimization method based on index atlases

A multi-objective parameter design method of rotavator working parameters is presented, which is a probability parameter optimization method based on index atlases. This method aims at multiple parameters sampling which has various distribution characteristics, and regards values of multiple evaluation indices as objectives. Then it selects proper geometric and physical parameters according to mathematical model calculations and distribution regulation of sampling values. Set \( X=(x_1, x_2, \ldots, x_n) \) as sample parameter and its probability distribution function is \( f(X) \), then the parameter optimization model is

\[
P(x_i) = P \left\{ \frac{R_{o}}{V_f} \max \{M\} f_i(X) \in R_s, \cdots, f_n(X) \in R_n \right\} \quad X \in f(X)
\]

Where, \( f_i(X) (i=1,2,\ldots,n) \) are objective functions of various design parameters and \( R_i (i=1,2,\ldots,n) \) are design objective
values of various objective functions. Based on a probability distribution of each design parameter, a set of reasonable structural design parameters are selected.

The Spacing model technique is used to non-dimensional operating of rotavator working parameters, all possible combinations of which are represented by limited space patterns. Also, the relationship between each working parameter and its performance is constructed, which is a new method for realizing optimization design of working parameters (Li, Jin and Ji, 2009).

Supposing that the working parameters of rotavators are mainly composed of forward travel velocity $V_f$, tillage blade rotation velocity $\omega$ and tillage depth $d$. Spacing model technique is used to define non-dimensional working parameters and establish spacing model.

$$U_0 = (V_f + \omega + d) / 3$$  \hspace{1cm} (14)

Then, the non-dimensional working parameters of $V_f$, $\omega$ and $d$ are

$$\begin{align*}
    r_1' &= V_f / U_0 \\
    r_2' &= \omega / U_0 \\
    r_3' &= d / U_0
\end{align*}$$  \hspace{1cm} (15)

Using equations (18) and (19), it can be gained

$$u_1 + u_2 + u_3 = 3$$  \hspace{1cm} (16)

Adopting $u_1$, $u_2$ and $u_3$ as the Cartesian coordinate system and using equation (20) to establish a geometric spacing model of rotavator operation parameters, its model is formed by triangle $H'K'R'$, as shown in Figure 4. The values range of each working parameter is

$$\begin{align*}
    0 \leq r_1' &\leq 3 \\
    0 \leq r_2' &\leq 3 \\
    0 \leq r_3' &\leq 3
\end{align*}$$  \hspace{1cm} (17)

To facilitate the description, triangle $H'K'R'$ is mapped to the two-dimensional $O$-$xy$ plane. Its coordinate mapping relationship is

$$\begin{align*}
    x &= 2r_1' / \sqrt{3} + r_2' / \sqrt{3} \\
    y &= r_3'
\end{align*}$$  \hspace{1cm} (18)

4.2 Optimization case of forward travel speed, rotation velocity of a rotavator

In this optimization case, the design parameters are mainly considered as forward travel speed $V_f$ and tillage blade rotation velocity $\omega$. Then there are two objective functions $f(\lambda)$ and $f(M)$ that can be defined by equation (1) and equation (12). Those design objective values depend on respectively the value of the pulverizing rate $\lambda_{set}$, from the literature (Guo, Zhou and Zhang, 2009), the value of $\lambda_{set}$ is 4 to 10, the pulverizing effect of soil is better and the value of requirement power $M_{set}$ is up to maximum. Then the $f(X)$ can be designed as uniform distribution function, therefore, the optimization model is shown as follows:

$$\begin{align*}
    P(V_f, \omega) = P\{f_1(X) \in \lambda_{set}, \max(\Sigma P_i \times V_i + \Sigma P_i \times V_i) \leq M_{set}\} \\
    X \in f(X)
\end{align*}$$  \hspace{1cm} (19)

The known design parameters for this optimization project are listed in Table 1.

Based on cultivation power, probability parameters design method is adopted to select a reasonable working velocity parameter of the rototiller. Using optimization model, i.e., equation (19), with equations (4) – (12), done in MATLAB program, the graphs of the mapping relationships between evaluation indices of power and pulverizing rate to working rotavator parameters separately are drawn, as shown in Figure 5 and Figure 6.
Table 1  Listing of the model input variables and their assigned values

<table>
<thead>
<tr>
<th>Description of the Model input variable</th>
<th>Notation</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil dynamics and strength parameters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angle of internal soil friction/(°)</td>
<td>( \phi )</td>
<td>30°</td>
</tr>
<tr>
<td>Soil bulk density/g · m(^{-3})</td>
<td>( \gamma )</td>
<td>2.8</td>
</tr>
<tr>
<td>Soil cohesion/kN · m(^{-2})</td>
<td>( C_s )</td>
<td>20</td>
</tr>
<tr>
<td>Soil-metal friction angle/(°)</td>
<td>( \delta )</td>
<td>32.06</td>
</tr>
<tr>
<td>Soil-metal adhesion/kN · m(^{-2})</td>
<td>( C_a )</td>
<td>8.12</td>
</tr>
<tr>
<td>Set depth of tillage/mm</td>
<td>( d )</td>
<td>200</td>
</tr>
<tr>
<td>Angle ( \beta )(°)</td>
<td>( \beta )</td>
<td>30</td>
</tr>
<tr>
<td>Angle ( \delta )(°)</td>
<td>( \delta )</td>
<td>30</td>
</tr>
<tr>
<td>Angle ( \rho )(°)</td>
<td>( \rho )</td>
<td>30</td>
</tr>
<tr>
<td>Rotavator parameter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor radius/mm</td>
<td>( r )</td>
<td>290</td>
</tr>
<tr>
<td>Width of blade/mm</td>
<td>( w )</td>
<td>40</td>
</tr>
<tr>
<td>Leg length of the blade</td>
<td>( L )</td>
<td>115.4</td>
</tr>
<tr>
<td>Bit length/mm</td>
<td>( L_b )</td>
<td>90</td>
</tr>
<tr>
<td>Span length of span/mm</td>
<td>( S )</td>
<td>130</td>
</tr>
<tr>
<td>Set kinematic parameter</td>
<td>( \lambda )</td>
<td>4–10</td>
</tr>
<tr>
<td>Range of forward travel speed/m · s(^{-1})</td>
<td>( V_{f\text{set}} )</td>
<td>0–1</td>
</tr>
<tr>
<td>Range of power shaft rotation velocity/r · min(^{-1})</td>
<td>( \omega_{\text{set}} )</td>
<td>0–1,000</td>
</tr>
<tr>
<td>Set requirement power of tillage blade/kW</td>
<td>( M_{\text{set}} )</td>
<td>≤ 0.185</td>
</tr>
</tbody>
</table>

Figure 5  Relationship between requirement power \( M \) Vs forward travel speed \( V_f \)

Figure 6  Relationship between pulverizing rate and cultivation velocity

Therefore, seen from Figure 5 and Figure 6, taking pulverizing rate values 4 to 10 and the requirement power equal to 185 W as design objectives, a geometric spacing model of rotavator operation parameters is sampled according to uniform distribution in the value range of each design parameter. Then, under the situations of all performance evaluation indices better than the designed objective value, the distribution pattern of each parameter sampling value is counted to draw a frequency histogram. Figure 7 shows the design histogram of working parameters of this rotary tool. Where, \( f(V_f) \) and \( f(\omega) \) are the probabilities better than objective design value respectively. From Figure 7, it can be deduced that with working parameters of \( \omega = 400 \) r/min, \( V_f = 0.9 \) m/s, power required by the rotavator tool is less and its pulverizing rate is higher in many working patterns with various working speeds. Therefore, for these soil and tillage demands, the optimization scheme for less requirement power and better soil pulverizing effect, are \( \omega = 400 \) r/min and \( V_f = 0.9 \) m/s, respectively.

![Histogram of working speeds of the rotavator with uniform probability distribution](image)

Moreover, other soil tillage tasks with other two type soil can be optimized with above-mentioned method, the
optimization results, constrained with maximum requirement power of a tillage blade $M_{max}$ no more than 0.185 kW and with preset-value of kinematic parameter $\lambda$ which is 4 to 10, are shown in Table 2.

<table>
<thead>
<tr>
<th>Soil classification</th>
<th>Soil bulk density /g $\cdot$ m$^{-3}$</th>
<th>Water content /%</th>
<th>Soil cohesion /kN $\cdot$ m$^{-2}$</th>
<th>Soil-metal adhesion /kN $\cdot$ m$^{-2}$</th>
<th>optimizing working parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>garden soil</td>
<td>$1.5 \times 10^{-3}$</td>
<td>50</td>
<td>60</td>
<td>1.41</td>
<td>rotation velocity of power shaft/ r $\cdot$ min$^{-1}$: 500</td>
</tr>
<tr>
<td>sand clay</td>
<td>$2.1 \times 10^{-3}$</td>
<td>30</td>
<td>10</td>
<td>1.01</td>
<td>forward velocity /m $\cdot$ s$^{-1}$: 0.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Depth of tillage /mm: 260</td>
</tr>
</tbody>
</table>

5 Conclusions

In this paper, an analytical power consumption model for the typical small rotavator with L-shaped blades was deduced. An analytical working parameters optimization algorithm based on power required by a rotavator fitted with L-shaped blades, which is a probability parameter optimization method based on index atlases, was proposed. In an agricultural rotavator equipped with typical L-shaped blades tilling a soil of tea gardens in east-southern of China, for example, in order to reduce the peak power requirement and to guarantee soil pulverizing effect, the power consumption optimization model involving main working parameters, such as power required by a blade, soil pulverization rate, forward travel speed, tillage blade rotational velocity and tillage depth, is carried out in conditions of a maximum tillage blade power requirement ($M_{max}$) no more than 0.185 kW and with a kinematic parameter $\lambda$ preset-value of 4 to 10, the results show that the optimized tillage blade rotation velocity is $\omega = 400$ r/min and forward travel speed $V_f = 0.9$ m/s. Moreover, other working speed optimization cases for various soil tilling methods with a small rotavator were also carried out, and the results show that the proposed models and methods are available.

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