

# EFFECTS OF FIBER LENGTH AND FIBER ORIENTATION DISTRIBUTIONS ON THE TENSILE STRENGTH OF SHORT-FIBER-REINFORCED POLYMERS

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## Abstract

*This paper presents an analytical method considering the effects of fiber length and fiber orientation distributions for predicting the tensile strength (TS) of short-fiber-reinforced polymers (SFRP). Two probability density functions are used for modelling the distributions of fiber length and fiber orientation. The strength of SFRP is derived as a function of fiber length and fiber orientation distribution taking into account the dependences of the ultimate fiber strength and the critical fiber length on the inclination angle and the effect of inclination angle on the bridging stress of oblique fibers. Then the effects of the mean fiber length, the most probable length (mode length), the critical fiber length, the mean fiber orientation, the most probable fiber orientation and the fiber orientation coefficient on the tensile strength of SFRP have been studied in detail. This model provides the necessary information to determine what fiber length distribution, what fiber orientation distribution and what interfacial adhesion are required to achieve a desired composite strength. The present theory is then applied to existing experimental results and the agreement is found to be satisfactory. © 1996 Elsevier Science Limited*

**Keywords:** composites, fiber length distribution, fiber orientation distribution, strength of SFRP

## 1 INTRODUCTION

Short-fiber-reinforced polymers (SFRP) are used increasingly as a structural material because they not only provide superior mechanical properties, but they can also be easily produced by the rapid, low-cost injection molding process. During the injection process of fiber-reinforced molded articles, the distributions of fiber length and fiber orientation are governed by a variety of factors. These include the original length and concentration of fibers, the gate design and the processing conditions.<sup>1–7</sup> It was pointed out that the mechanical properties of any material

depend on the details of its structure.<sup>8</sup> For example, the stiffness and strength of machine-made paper have been shown to depend on the fiber length and fiber orientation distributions.<sup>8–10</sup> It has also been shown that the mechanical properties of injection molded SFRP depend critically on the fiber length distribution (FLD) and the fiber orientation distribution (FOD).<sup>7,11–15</sup> In order to obtain the high performance injection molded short-fiber-reinforced thermoplastics, it is therefore of great importance to study the effects of the FLD and the FOD on the mechanical properties of injection molded SFRP. As a part of our research project, we present a theoretical study of the tensile strength (TS) of SFRP by considering the effects of the FLD and the FOD.

The modified rule of mixtures (MROM) is often used to predict the tensile strength of short-fiber composites by assuming a perfect interfacial bond between fibers and matrix.<sup>16–23</sup> The formula of the MROM is given by:

$$\sigma_{cu} = \chi_1 \chi_2 V_f \sigma_{fu} + V_m \sigma_m \quad (1)$$

where  $\chi_1$  and  $\chi_2$  are, respectively, the fiber orientation and fiber length factors, and the product of  $\chi_1$  and  $\chi_2$ , i.e.  $\chi_1 \chi_2$ , is the fiber efficiency factor for the strength of the composite;  $\sigma_{cu}$  and  $\sigma_{fu}$  are the ultimate strength of the composite and fiber, respectively;  $V_f$  and  $V_m$  denote the volume fraction of the fiber and matrix; and  $\sigma_m$  is the matrix stress at the failure of the composite. For unidirectional discontinuous composites  $\chi_1 = 1$  and  $\chi_2 < 1$ . If the fiber length is uniform and equals  $L$ , then

$$\chi_2 = L/(2L_c) \text{ for } L < L_c \quad (2)$$

$$\chi_2 = 1 - L_c/(2L) \text{ for } L \geq L_c \quad (3)$$

where  $L_c$ , the critical fiber length, is given by  $L_c = r_f \sigma_{cu} / \tau_i$ , where  $\tau_i$  and  $r_f$  are the interfacial shear stress between matrix and fibers and the fiber radius, respectively. If the fiber length is not uniform, eqns (2) and (3) must be modified. Kelly and Tyson<sup>22</sup> put forward a model considering the effect of fibers,

shorter and longer than the critical fiber length. This model can be given by:<sup>17</sup>

$$\sigma_{cu} = \sum_{L_i=L_{min}}^{L_c} V_i \sigma_{fu} L_i / (2L_c) + \sum_{L_j=L_c}^{L_{max}} V_j \sigma_{fu} [1 - L_c / (2L_j)] + V_m \sigma_m \quad (4)$$

The first and second terms take into account the contributions of fibers with sub-critical length shorter than  $L_c$  and of fibers with super-critical length longer than  $L_c$ , respectively. However, the contribution of the fiber orientation is not considered. This model is modified by the addition of an orientation factor to the first two terms of eqn (4), but the orientation factor is only fitted empirically:<sup>7,23</sup>

$$\sigma_{cu} = \chi_1 \left\{ \sum_{L_i=L_{min}}^{L_c} V_i \sigma_{fu} L_i / (2L_c) + \sum_{L_j=L_c}^{L_{max}} V_j \sigma_{fu} [1 - L_c / (2L_j)] \right\} + V_m \sigma_m \quad (5)$$

The critical damage zone model of Fukuda and Chou was also used to predict the tensile strength of the SFRP.<sup>24</sup> This model goes one step further by considering the distribution function of both the fiber length and the fiber orientation. However, the generalized formulae of both the fiber length distribution function and the fiber orientation distribution function are not given, thus the effects of the fiber length and orientation distributions on the strength of SFRP cannot be studied; the dependences of the ultimate fiber strength and the critical fiber length on the inclination angle  $\theta$ , which must be considered,<sup>16,25,26</sup> are not taken into account.

In the present paper two probability density functions are used for modelling the fiber length and orientation distributions respectively, which have been used with the aim of predicting the elastic properties of SFRP.<sup>12,13,27</sup> The strength of SFRP is then derived as a function of the fiber length and fiber orientation distributions by considering the dependences of the ultimate fiber strength and the critical fiber length on the inclination angle and the effect of inclination angle on the bridging stress of oblique fibers. The effects of the fiber length distribution and the fiber orientation distribution on the tensile strength of composites are discussed in detail. Finally, the present theory is applied to existing experimental results.<sup>19</sup>

## 2 THEORY

### 2.1 Fiber length distribution (FLD)

During the extrusion and injection molding processes, the shear stresses exerted by the screw or ram will break the fibers and result finally in a fiber length

distribution with an asymmetric character with a tail at long fiber lengths.<sup>5,12,13,27,28</sup> The mechanical properties of SFRP are undoubtedly related to this distribution. The fiber length distribution can be described with a probability density function. Let us define the fiber length probability density function,  $f(L)$ , so that  $f(L)dL$  and  $F(L)$  are the probability density that the length of fiber is between  $L$  and  $L + dL$  and the probability that the length of a fiber is less than or equal to  $L$ , respectively. Then the relationship of  $f(L)$  and  $F(L)$  is:

$$F(L) = \int_0^L f(x) dx \quad \text{and} \quad \int_0^\infty f(x) dx = 1 \quad (6)$$

A two-parameter Weibull distribution function has been proposed for modelling the fiber length distribution and was shown to be effective in describing the density distribution for short-glass-fiber-reinforced polypropylene.<sup>13</sup>

$$f(L) = (m/n)(L/n)^{m-1} \exp[-(L/n)^m] \quad \text{for } L > 0 \quad (7)$$

where  $m$  and  $n$  are shape parameters. Another form of Weibull distribution, i.e. the so-called Tung distribution has been given:<sup>27</sup>

$$f(L) = abL^{b-1} \exp(-aL^b) \quad \text{for } L > 0 \quad (8)$$

where  $a$  and  $b$  are scale and shape parameters, respectively. Inserting  $b = m$  and  $a = n^{-m}$  into eqn (8), it then becomes the same as eqn (7). Equation (8) has also been used successfully for describing the fiber length distribution in short-glass-fiber-reinforced polyamide.<sup>27</sup> In the present study, therefore, we adopt this probability density function and prefer the form of eqn (8) as the fiber length distribution function of SFRP since it appears simple. The cumulative distribution function,  $F(L)$ , can be obtained by combining eqns (6) and (8):

$$F(L) = 1 - \exp(-aL^b) \quad \text{for } L > 0 \quad (9a)$$

Therefore, the percentage,  $\alpha$ , of fibers with lengths shorter than  $L_c$  can be evaluated by:

$$\alpha = 1 - \exp(-aL_c^b) \quad (9b)$$

From eqn (8) we can get the mean fiber length (i.e. the number average fiber length):

$$L_{mean} = \int_0^\infty Lf(L) dL = a^{-1/b} \Gamma(1/b + 1) \quad (10)$$

where  $\Gamma(x)$  is the gamma function. The most probable length (mode length),  $L_{mod}$ , can be obtained by differentiating eqn (8) and letting the resultant equation be equal to zero:

$$L_{mod} = [1/a - 1/(ab)]^{1/b} \quad (11)$$

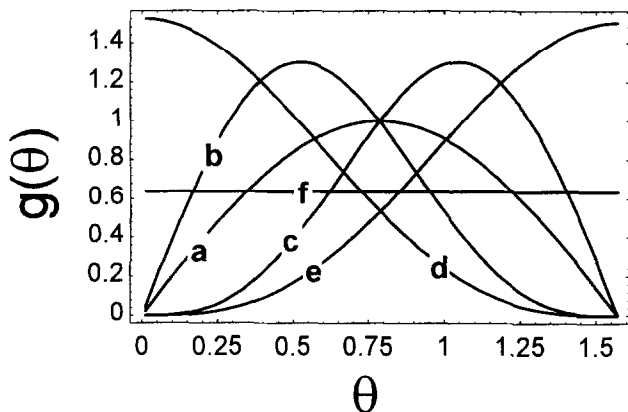
**2.2 Fiber orientation distribution (FOD)**

During extrusion compounding and injection molding processes, progressive and continuous changes in fiber orientation throughout the molded components take place. The changes are related in a complex way to the size and concentration of fibers, the flow behavior of melted polymer matrix, the mold cavity and the processing conditions. An orientation distribution generally requires a three-dimensional description. However, when investigating the effect of the fiber orientation angle on the strength of short-fiber composites, only the angle,  $\theta$ , between the fiber axis and the loading direction needs to be considered.<sup>19,24,29</sup> For example, the loading direction can be placed in the direction parallel to the mold fill direction, then this fiber orientation angle is the inclination angle or the out-of-plane angle, as defined previously,<sup>30-32</sup> respectively. A fiber orientation distribution function representing the inclination angle must have the property such that the variation of its function's shape parameters is able to describe a change from a unidirectional distribution to a random distribution.

Let us define a fiber orientation density function  $f(\theta)$  such that  $f(\theta)d\theta$  is the probability density that the orientation of fiber is between  $\theta$  and  $\theta + d\theta$ . With this assumption, Xia *et al.*<sup>12</sup> proposed a two-parameter exponential function to describe the fiber orientation distribution in the injection molded specimens. Similar to this definition, the fiber orientation distribution function is given as follows:

$$g(\theta) = \frac{\{\sin(\theta)\}^{2p-1}\{\cos(\theta)\}^{2q-1}}{\int_{\theta_{\min}}^{\theta_{\max}} \{\sin(\theta)\}^{2p-1}\{\cos(\theta)\}^{2q-1} d\theta} \quad (12)$$

where  $p$  and  $q$  are the shape parameters which can be used to determine the shape of the distribution curve, and  $p \geq 1/2$  and  $q \geq 1/2$ . Also,  $0 \leq \theta_{\min} \leq \theta \leq \theta_{\max} \leq \pi/2$ .



**Fig. 1.** Fiber orientation distribution curves for: (a)  $p = q = 1$ ; (b)  $p = 1, q = 2$ ; (c)  $p = 2, q = 1$ ; (d)  $p = 1/2, q = 2$ ; (e)  $p = 2, q = 1/2$ ; (f)  $p = q = 1/2$ .

Differentiating eqn (12) and letting the resultant equation be zero we get:

$$\theta_{\text{mod}} = \arctan\{[(2p - 1)/(2q - 1)]^{1/2}\} \quad (13)$$

Equation (13) represents the most probable fiber orientation angle. When  $p = q = 1$ ,  $\theta_{\text{mod}} = \pi/4$ ; when  $p = 1$  and  $q > 1$ , then  $\theta_{\text{mod}} > \pi/4$ ; when  $p > 1$  and  $q = 1$ , then  $\theta_{\text{mod}} > \pi/4$ ; when  $p = 1/2$ ,  $\theta_{\text{mod}} = 0$ ; when  $q = 1/2$ , then  $\theta_{\text{mod}} = \pi/2$ ; when  $p = q = 1/2$ , then there is no  $\theta_{\text{mod}}$  and the fibers distribute randomly; the corresponding fiber orientation distribution curves are shown in Fig. 1. When  $p = 1/2$ , large  $q$  (e.g. 100) indicates that fibers have a major preferential alignment parallel to the  $\theta = 0$  direction; when  $q = 1/2$ , large  $p$  (e.g. 100) expresses that fibers have a major preferential orientation normal to the  $\theta = 0$  direction; this is shown in Table 2. So, all the cases of fiber orientation distribution are included in eqn (12). Thus eqn (12) is a suitable probability density function for describing the fiber orientation distribution and will be used in the present study.

The mean fiber orientation,  $\theta_{\text{mean}}$ , can be derived from eqn (12) as follows:

$$\theta_{\text{mean}} = \int_{\theta_{\min}}^{\theta_{\max}} \theta g(\theta) d\theta \quad (14)$$

The fiber orientation coefficient,  $f_\theta$ , can be defined as follows:<sup>16,29</sup>

$$f_\theta = 2 \int_{\theta_{\min}}^{\theta_{\max}} g(\theta) \cos^2(\theta) d\theta - 1 \quad (15)$$

When  $f_\theta = -1$ , all fibers lie perpendicular to the loading direction;  $f_\theta = 0$  corresponds to a random distribution in the angle  $\theta$ ;  $f_\theta = 1$  implies all fibers are aligned parallel to the loading direction.

The cumulative distribution function of fiber orientation is then given by:

$$G(\theta) = \frac{\int_{\theta_{\min}}^{\theta} g(\theta) d\theta}{\int_{\theta_{\min}}^{\theta_{\max}} \{\sin(\theta)\}^{2p-1}\{\cos(\theta)\}^{2q-1} d\theta} \quad (16)$$

**2.3 Bridging stress of fibers**

When an applied load is exerted on a short-fiber-reinforced polymer, the interfacial shear stress between fibers and matrix will increase with the applied stress.<sup>33</sup> In order to estimate the force required to break the composite at some random cross-section, the bridging stress of fibers across the

failure plane needs to be evaluated<sup>16</sup> and the single fiber bridging case will be considered first.

When a fiber crosses the crack plane and the fiber orients parallel to the normal of the crack plane or the applied load,  $F$  (see Fig. 2(a)), then the bridging stress,  $\sigma_f$ , of the fiber across the crack is given by:

$$\sigma_f = L_s 2\pi r_f \tau_i / (\pi r_f^2) = 2L_s \tau_i / r_f \text{ for } L_s < L_c/2 \quad (17a)$$

$$\sigma_f = \sigma_{fu} \text{ for } L_s \geq L_c/2 \quad (17b)$$

where  $L_s$  is the length of shorter embedded fiber segment and  $\tau_i$  denotes the interfacial shear stress which we assume to be constant. However, when a fiber crosses obliquely with a crack plane (see Fig. 2(b)), then the bridging stress,  $\sigma_{f\theta}$ , can be given by:<sup>34-38</sup>

$$\sigma_{f\theta} = 2L_s (\tau_i / r_f) \exp(\mu\theta) \text{ for } L_s < L_{c\theta}/2 \quad (18)$$

where  $\theta$  is the angle between the fiber and the crack plane normal;  $\mu$  is the snubbing friction coefficient between fiber and matrix at the crossing point, which has been defined elsewhere;<sup>34,35</sup> and  $L_{c\theta}$  denotes the critical fiber length for an obliquely crossed fiber. When  $L_s \geq L_{c\theta}/2$ , the bridging stress of the oblique fiber is then given by:<sup>16,25</sup>

$$\sigma_{f\theta} = \sigma_{fu\theta} \text{ for } L_s \geq L_{c\theta}/2 \quad (19)$$

where  $\sigma_{fu\theta}$  denotes the fracture stress of the oblique fibers, i.e. the inclined tensile strength of fibers.<sup>26</sup> Since the flexural stresses cause an apparent loss of fiber strength, then the fracture strength for the oblique fiber is reduced.<sup>16,25</sup> Furthermore, the pull-out test of oblique steel fibers also indicated that the inclined tensile strength of oblique steel fibers is

reduced significantly and decreases with the increase of inclination angle. So the introduction of the inclined tensile strength of fibers can undoubtedly help to predict better the mechanical behavior of short-fiber composites. If the fibers are brittle (e.g. glass fibers, etc.), then  $\sigma_{fu\theta}$  can be expressed using the following formula:<sup>16,25</sup>

$$\sigma_{fu\theta} = \sigma_{fu} [1 - A \tan(\theta)] \quad (20)$$

where  $A$  is a constant for a specific fiber/matrix system. Obviously, there will be a maximum fiber orientation angle for  $\sigma_{fu\theta} \geq 0$ :

$$\theta_{\max} = \arctan(1/A) \quad (21)$$

When  $\theta \geq \theta_{\max}$ , then  $\sigma_{fu\theta} = 0$ . With the consideration of the snubbing friction effect and the fiber flexural effect the critical length of oblique fibers can be derived from eqns (18)–(20):

$$L_{c\theta} = L_c [1 - A \tan(\theta)] / \exp(\mu\theta) \quad (22)$$

If the effect of snubbing friction is neglected, i.e.  $\mu = 0$ , then eqn (22) becomes:

$$L_{c\theta} = L_c [1 - A \tan(\theta)] \quad (23a)$$

Equation (23a) is the same as that given in previously.<sup>16</sup> If the effect of fiber flexural is neglected, i.e.  $A = 0$ , then eqn (22) becomes:

$$L_{c\theta} = L_c / \exp(\mu\theta) \quad (23b)$$

Equation (23b) is the same as that given elsewhere.<sup>37,38</sup> If both the snubbing friction effect and the fiber flexural effect are neglected, i.e.  $\mu = 0$  and  $A = 0$ , then eqn (22) becomes:

$$L_{c\theta} = L_c \quad (23c)$$

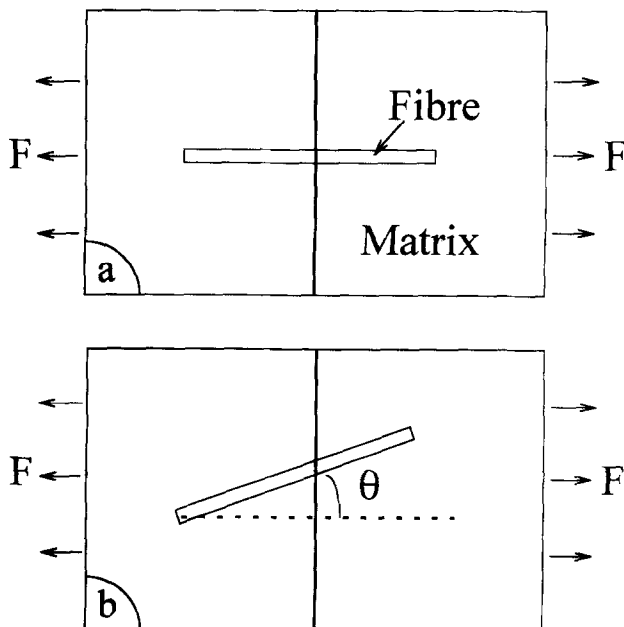
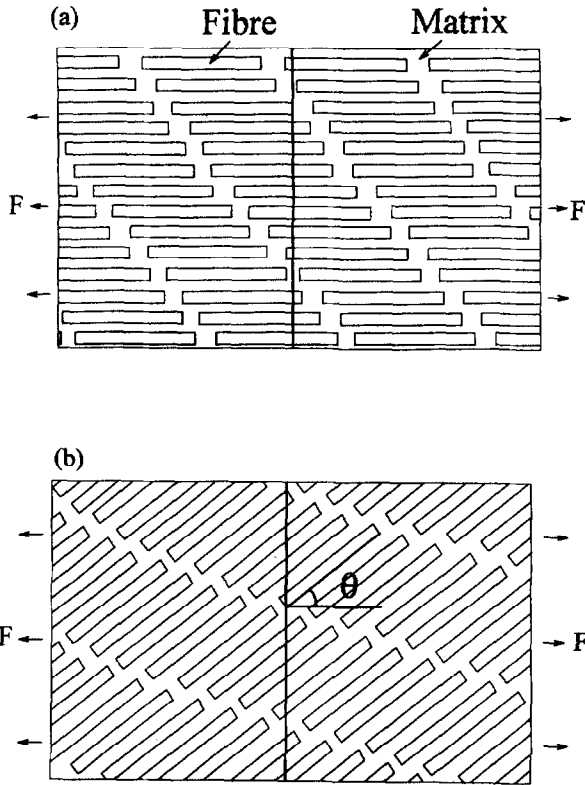


Fig. 2. Schematic drawing of a fiber across a crack: (a) the fiber which orients parallel to the crack plane normal; (b) the fiber which crosses obliquely with the crack plane.

2.4 Average fiber stress in the aligned composites

First, we consider an aligned fiber composite. The fibers all have the same length,  $L$ , and diameter,  $d$ , and are perfectly aligned in the direction of the applied load,  $F$ . It is assumed that the fibers distribute uniformly in the composites. We will estimate the stress required to break the composite at some random cross-section, such as that shown in Fig. 3(a). The fiber length of the shorter fiber segment across the failure plane ranges from 0 to  $L/2$ . When the fiber length of the shorter segment across the crack plane is less than  $L_c/2$ , then the fibers will debond fully and pull out against the shear stress,  $\tau_i$ , at the failure of composites. Otherwise, the fibers will break.

When the fiber length,  $L$ , is less than the critical fiber length,  $L_c$ , then the fiber will be pulled out due to the fact that the fiber length of the shorter segment is certainly less than  $L_c/2$ . The fiber length of the shorter segment in the matrix crossing the crack plane varies from 0 to  $L/2$  and the average fiber length of shorter segment across the crack plane is  $L/4$  because of the assumption that the fibers distribute uniformly



**Fig. 3.** Schematic drawing of a crack which has developed across fibers: (a) the fibers aligned in the direction perpendicular to the cross-section; (b) the fibers crossed obliquely with the cross-section.

in the matrix. Then the average bridging stress of fibers across the crack plane is:

$$\sigma_f = (L/4)2\pi r_f \tau_i / (\pi r_f^2) = L\tau_i/d \quad (24)$$

thus

$$\sigma_f = \sigma_{fu}L/(2L_c) \quad (25)$$

where

$$\sigma_{fu} = L_c \tau_i / r_f \quad (26)$$

When the fiber length,  $L$ , is greater than  $L_c$ , then the shorter segment of fibers will be pulled out if the shorter segment length is less than  $L_c/2$ , and the shorter segment length of fibers will vary from 0 to  $L_c/2$ . Thus, the average length of shorter segments is  $L_c/4$  and the corresponding fraction of fibers is  $L_c/L$ . If the shorter segment length of fibers is greater than  $L_c/2$ , then the fibers will not be pulled out and will break at the failure of composites and the fraction of fibers is  $1 - L_c/L$ . Thus the average bridging stress of fibers with a length greater than  $L_c$  can be given by:

$$\begin{aligned} \sigma_f &= [(L_c/L)(L_c/4)2\pi r_f \tau_i + (1 - L_c/L)\pi r_f^2 \sigma_{fu}] / (\pi r_f^2) \\ &= \sigma_{fu}(1 - L_c/(2L)) \end{aligned} \quad (27)$$

It is easily seen that eqns (26) and (27) are the same as eqns (2) and (3). This indicates that our derivation process is reasonable. In the above derivation, for simplicity we neglect the stresses across the fiber ends,

and the effect of the crack diversion for the very short embedded segment case which was considered by Piggott.<sup>16</sup>

Secondly, we consider composites with the fibers oriented obliquely at an angle  $\theta$  with the crack plane, a part of which is shown schematically in Fig. 3(b). Similar to the above derivation, we get the average bridging stress of fibers crossing the crack plane as follows:

$$\begin{aligned} \sigma_{f\theta} &= (L/4)2\pi r_f \tau_i \exp(\mu\theta) / (\pi r_f^2) \\ &= \sigma_{fu}L \exp(\mu\theta) / (2L_c) \text{ for } L < L_{c\theta} \end{aligned} \quad (28)$$

$$\begin{aligned} \sigma_{f\theta} &= \{(L_{c\theta}/L)(L_{c\theta}/4)2\pi r_f \tau_i \exp(\mu\theta) \\ &\quad + (1 - L_{c\theta}/L)\sigma_{fu\theta}\pi r_f^2\} / (\pi r_f^2) \\ &= \sigma_{fu\theta}(1 - L_{c\theta}/(2L)) \text{ for } L \geq L_{c\theta} \end{aligned} \quad (29)$$

When  $\theta$  equals zero, eqns (28) and (29) are the same as eqns (2) and (3).

## 2.5 Strength of SFRP

Based on the above discussion it becomes possible to derive the strength of SFRP. The composite strength can be evaluated in two different ways.

### 2.5.1 Method 1

We assume that the fibers distribute uniformly in the composite, then the failure strength of any cross-section of composite can represent the composite strength. If  $N$  denotes the total number of fibers in the composites,  $N_i$  is the number of fibers with a length from  $L$  to  $L + dL$  and the orientation angle from  $\theta$  to  $\theta + d\theta$ .  $f(L)$  and  $g(\theta)$  are the functions of the fiber length distribution and the fiber orientation distribution respectively, where  $f(L)$  and  $g(\theta)$  are independent. Then:

$$N_i/N = f(L)g(\theta)dLd\theta \quad (30)$$

The volume subfraction,  $V_i$ , of the fibers of a length from  $L$  to  $L + dL$  and an orientation angle from  $\theta$  to  $\theta + d\theta$  can be obtained:

$$\begin{aligned} V_i &= V_f[(N_i L \pi r_f^2) / (N L_{\text{mean}} \pi r_f^2)] \\ &= V_f[(N_i L) / (N L_{\text{mean}})] \end{aligned} \quad (31)$$

where  $V_f$  is the volume fraction of fibers in the composites and  $L_{\text{mean}}$  is the mean fiber length of all the fibers in the composites. The composite strength is contributed by all the fibers of length from  $L_{\text{min}}$  to  $L_{\text{max}}$  and the orientation angle from  $\theta_{\text{min}}$  to  $\theta_{\text{max}}$  and the matrix, then:

$$\sigma_{cu} = \sum_{\theta=\theta_{\text{min}}}^{\theta_{\text{max}}} \sum_{L=L_{\text{min}}}^{L_{\text{max}}} V_i \sigma_{f\theta} + \sigma_m V_m \quad (32)$$

Combining eqns (30)–(32) and replacing the summation by the integral, we get:

$$\sigma_{cu} = V_f \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \int_{L_{\text{min}}}^{L_{\text{max}}} f(L)g(\theta)(L/L_{\text{mean}})\sigma_{f\theta} dLd\theta + \sigma_m V_m \quad (33)$$

Substituting eqns (20), (22), (28) and (29) into eqn (33), we get the strength of SFRP:

$$\begin{aligned} \sigma_{cu} = V_f & \left[ \int_{\theta_{\min}}^{\theta_{\max}} \int_{L_{\min}}^{L_{c0}} f(L)g(\theta)(L/L_{\text{mean}})\sigma_{fu}(L/(2L_c)) \right. \\ & \times \exp(\mu\theta) dLd\theta + \int_{\theta_{\min}}^{\theta_{\max}} \int_{L_{c0}}^{L_{\max}} f(L)g(\theta)(L/L_{\text{mean}}) \\ & \times \sigma_{fu}(1 - A \tan(\theta))(1 - L_c(1 - A \tan(\theta))) \\ & \left. / (2L \exp(\mu\theta)) dLd\theta \right] + \sigma_m V_m \end{aligned} \quad (34)$$

Equation (34) can be rewritten in the following form:

$$\sigma_{cu} = \chi_1 \chi_2 V_f \sigma_{fu} + \sigma_m V_m \quad (35)$$

where

$$\begin{aligned} \chi_1 \chi_2 = & \int_{\theta_{\min}}^{\theta_{\max}} \int_{L_{\min}}^{L_{c0}} f(L)g(\theta)(L/L_{\text{mean}})(L/(2L_c)) \\ & \times \exp(\mu\theta) dLd\theta + \int_{\theta_{\min}}^{\theta_{\max}} \int_{L_{c0}}^{L_{\max}} f(L)g(\theta) \\ & \times (L/L_{\text{mean}})(1 - A \tan(\theta)) \\ & \times (1 - L_c(1 - A \tan(\theta)))/(2L \exp(\mu\theta)) dLd\theta \end{aligned} \quad (36)$$

The larger the value of  $\chi_1 \chi_2$ , the higher is the composite strength. If  $\theta = 0$ ,  $\chi_1$  should be equal to 1 and it is the case for unidirectionally aligned short-fiber composites. Then the fiber length factor is:

$$\begin{aligned} \chi_2 = & \int_{L_{\min}}^{L_c} [L^2/(2L_c L_{\text{mean}})] f(L) dL \\ & + \int_{L_c}^{L_{\max}} (L/L_{\text{mean}})[1 - L_c/(2L)] f(L) dL \end{aligned} \quad (37)$$

### 2.5.2 Method 2

Suppose a rectangular shaped specimen with the lengths of the three mutually perpendicular edges denoted by  $a$ ,  $b$  and  $c$  is considered. The  $c$  axis is chosen to be parallel to the loading direction. The volume of the specimen is:

$$V = abc \quad (38)$$

In accordance with the definition, the fiber volume fraction is given by:

$$V_f = N A_f L_{\text{mean}} / V \quad (39)$$

where  $A_f$  is the fiber cross-sectional area. The length of the projection of fibers of length  $L$  and angle  $\theta$  with

respect to the loading direction on the  $c$  axis (i.e. loading direction) is:

$$L_p = L \cos(\theta) \quad (40)$$

Thus the average number of fibers of length from  $L$  to  $L + dL$  and an angle from  $\theta$  to  $\theta + d\theta$ , which cross an arbitrary section in the specimen, can be given by:

$$N_c = N_i L_p / c \quad (41)$$

where  $N_i$  is the number of fibers of length from  $L$  to  $L + dL$  and an angle from  $\theta$  to  $\theta + d\theta$  as that given by eqn (30). Then the composite strength can be obtained:

$$\sigma_{cu} = (1/A_c) \sum_{\theta=\theta_{\min}}^{\theta_{\max}} \sum_{L=L_{\min}}^{L_{\max}} N_c \sigma_{f\theta} A_{f\theta} + \sigma_m V_m \quad (42)$$

where  $A_c$  is the cross-sectional area of the specimen and equals  $ab$ ;  $A_{f\theta}$  is the cross-sectional area of oblique fibers of an angle  $\theta$  with respect to the  $c$  axis (i.e. the loading direction) and equals  $A_f / \cos(\theta)$ . Substituting eqns (30), (38)–(41) into eqn (42) and replacing the summation sign with the integral sign, we finally obtain:

$$\sigma_{cu} = V_f \int_{\theta_{\min}}^{\theta_{\max}} \int_{L_{\min}}^{L_{\max}} f(L)g(\theta)(L/L_{\text{mean}})\sigma_{f\theta} dLd\theta + \sigma_m V_m \quad (43)$$

It can be seen that eqns (33) and (43) are the same, therefore methods 1 and 2 give identical results.

## 3 RESULTS AND DISCUSSION

Initially, unidirectional composites ( $\chi_1 = 1$ ) are considered. In this case the effect of fiber length distribution on the strength of composites can be studied by using eqn (37).

From eqn (37) the value of  $\chi_2$  can be estimated if  $L_{\min}$ ,  $L_{\max}$ ,  $L_c$  and  $f(L)$  are given, then the tensile strength of short-fiber composites can be evaluated with eqn (35). Assuming  $L_{\text{mod}} = 0.2$  mm and a set of the values of the parameter  $b$ , then we get a set of values of the parameter  $a$  from eqn (11), the mean fiber length  $L_{\text{mean}}$  from eqn (10), and the probability density function  $f(L)$  from eqn (8), respectively. The percentage of the fibers with a length shorter than  $L_c$ , can be evaluated with eqn (9b). Assuming  $L_{\min}$  and  $L_{\max}$  to be 0 and  $\infty$  mm, respectively, and  $L_c = 0.2, 0.4$  and  $0.8$  mm, then the calculated results are shown in Figs 4 and 5. The percentage of the fibers with subcritical fiber length decreases dramatically with the increase of mean fiber length at small mean fiber lengths (in the vicinity of  $L_c$ ) and reaches gradually a plateau level as  $L_{\text{mean}}$  increases at large mean fiber lengths (greater than about  $5L_c$ ) (see Fig. 4). Since the supercritical fibers with lengths longer than  $L_c$  would have a larger contribution to the composite strength than that of subcritical fibers, the value of  $\chi_2$  increases

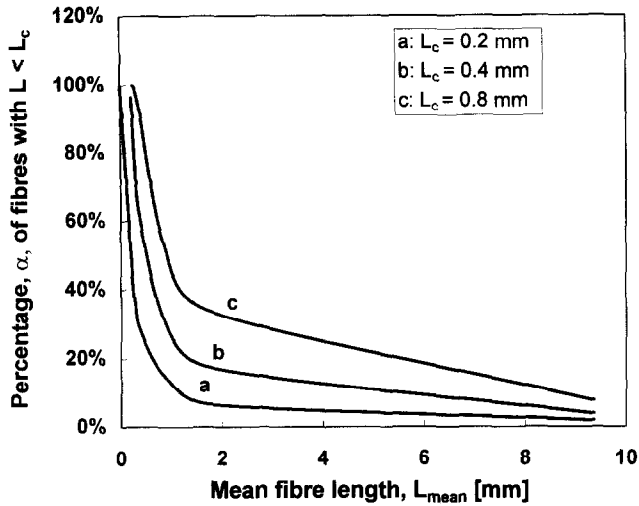


Fig. 4. Variation of the percentage,  $\alpha$ , of fibers with  $L < L_c$  as mean fiber length,  $L_{mean}$ , varies for the cases of different  $L_c$  ( $L_{mod} = 0.2$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

rapidly with the increase of mean fiber length,  $L_{mean}$ , at small mean fiber lengths (in the vicinity of  $L_c$ ) and approaches gradually a plateau level at large mean fiber lengths (greater than about  $5L_c$ ) (see Fig. 5). Thus the tensile strength of short-fiber composites increases rapidly as the the mean fiber length,  $L_{mean}$ , increases for the cases of small mean fiber lengths and approaches a plateau level as  $L_{mean}$  increases for the cases of large mean fiber lengths. This is consistent with the existing experimental results.<sup>39</sup> Moreover, the smaller critical fiber length corresponds to the lower content of fibers with subcritical length (see Fig. 4) and hence to the higher value of  $\chi_2$  (see Fig. 5). So the smaller critical fiber length brings about a higher composite strength. The effect of the critical fiber length on the value of fiber length factor,  $\chi_2$ , and hence on the composite strength is further shown in

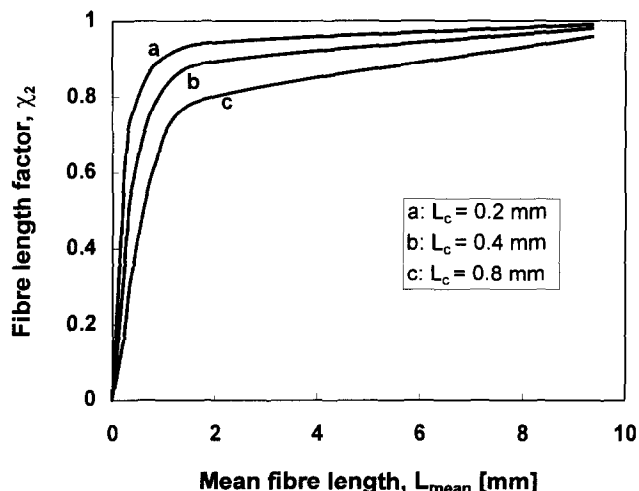


Fig. 5. Effects of mean fiber length,  $L_{mean}$ , and critical fiber length,  $L_c$ , on fiber length factor,  $\chi_2$ , for the case of unidirectional composites ( $L_{mod} = 0.2$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

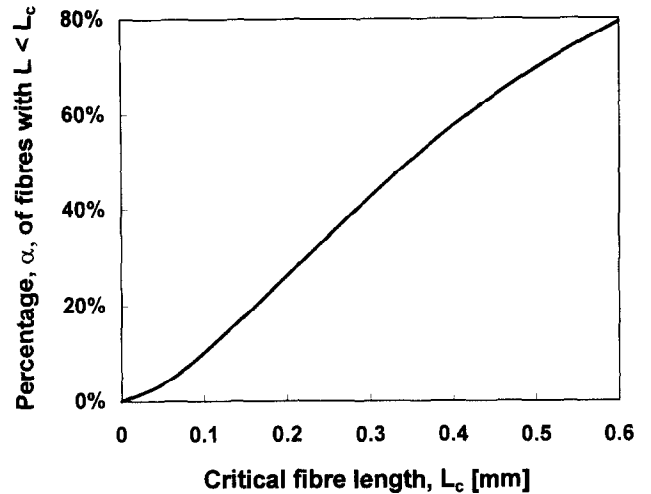


Fig. 6. Variation of the percentage,  $\alpha$ , of fibers with  $L < L_c$  as critical fiber length,  $L_c$ , varies ( $L_{mean} = 0.4$  mm,  $L_{mod} = 0.213$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

Figs 6 and 7, where  $L_{min} = 0$ ,  $L_{max} = \infty$ ,  $L_{mean} = 0.4$  mm,  $L_{mod} = 0.213$  mm for various  $L_c$ . It can easily be seen that the percentage of fibers with subcritical length increases with the critical fiber length (see Fig. 6) and hence  $\chi_2$  decreases with the increase of critical fiber length (see Fig. 7); therefore, the tensile strength of SFRP decreases with the critical fiber length. This is consistent with the experimental results.<sup>17</sup> Due to the fact that the critical fiber length is inversely proportional to the interfacial adhesion strength, namely  $L_c = r_f \sigma_{cu} / \tau_i$ , the strength of SFRP increases with the increase of interfacial adhesion strength.

Assuming  $L_{min}$ ,  $L_{max}$  and  $L_{mean}$  to be 0,  $\infty$  and 0.4, and  $L_c$  to be 0.2, 0.4 and 0.8 mm, respectively, the calculated results for the percentage,  $\alpha$ , of fibers with subcritical lengths and the fiber length factor,  $\chi_2$ , are presented in Figs 8 and 9, respectively. From Fig. 8 it

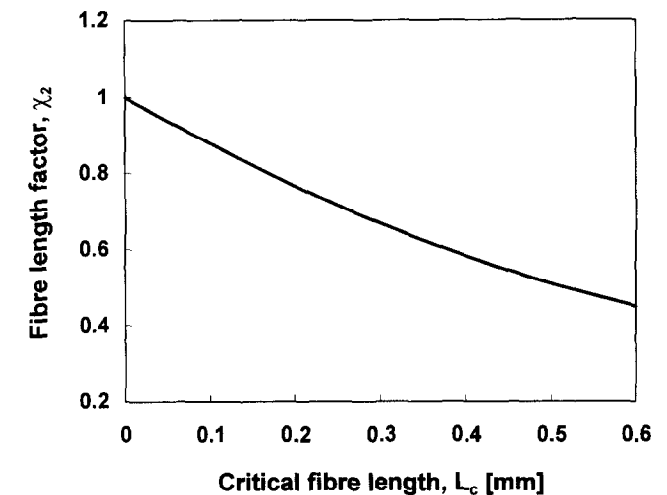


Fig. 7. Effect of critical fiber length,  $L_c$ , on fiber length factor,  $\chi_2$ , for the case of unidirectional composites ( $L_{mean} = 0.4$  mm,  $L_{mod} = 0.213$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

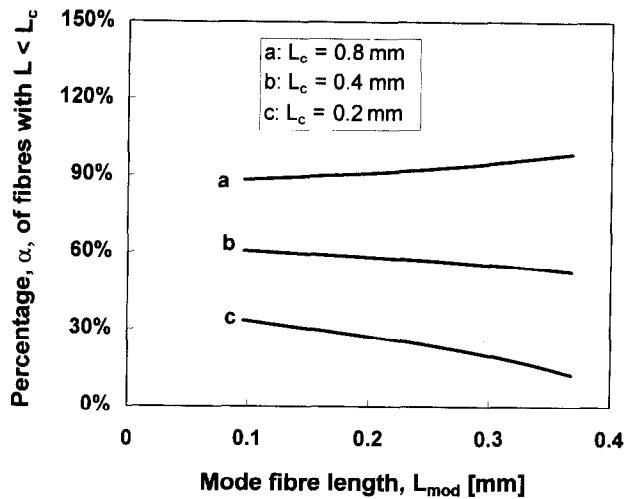


Fig. 8. Variation of the percentage,  $\alpha$ , of fibers with  $L < L_c$  as mode fiber length,  $L_{mod}$ , varies for different  $L_c$  ( $L_{mean} = 0.4$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

can be seen that the content of the fibers with subcritical fiber length decreases with the increase of  $L_{mod}$  for smaller critical fiber length, i.e.  $L_c = 0.2$  and  $0.4$  mm. However, the content of the fibers with subcritical fiber length increases with the increase of  $L_{mod}$  for larger critical fiber length, i.e.  $L_c = 0.8$  mm. Figure 9 shows that  $\chi_2$  decreases with  $L_{mod}$ , thus the tensile strength of short-fiber composites decreases with the mode fiber length. However, the effect of the mode fiber length on the tensile strength is small, especially for the case of  $L_c < L_{mean}$ , e.g.  $L_c = 0.2$  mm  $<$   $L_{mean} = 0.4$  mm, this effect is very small (see Fig. 9).

When the values of  $L_{mean}/L_c$  and  $L_{mod}/L_c$  are given, for example  $L_{mean}/L_c = 2$  or  $0.5$  and  $L_{mod}/L_c = 1.065$  or  $0.266$ , respectively, the calculated results are given in Table 1, where  $L_{min} = 0$  and  $L_{max} = \infty$  mm. An

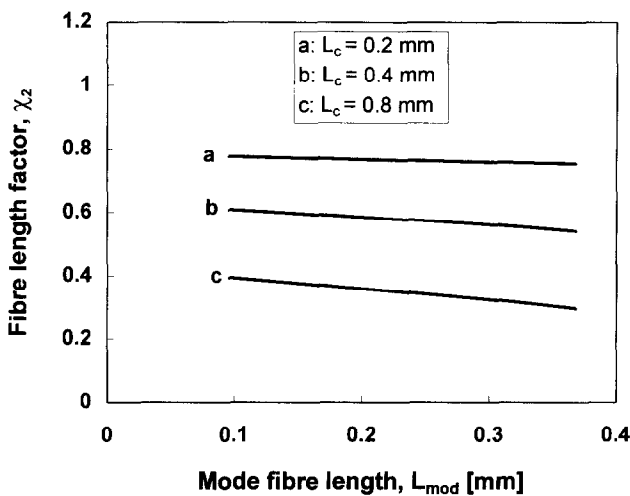


Fig. 9. Effects of mode fiber length,  $L_{mod}$ , and critical fiber length,  $L_c$ , on fiber length factor,  $\chi_2$ , for the case of unidirectional composites ( $L_{mean} = 0.4$  mm,  $L_{min} = 0$  mm and  $L_{max} = \infty$  mm).

Table 1. Parameters  $b$  and  $a$ , mean fiber length, mode fiber length, critical fiber length, the content of fibers with a subcritical fiber length and fiber length factor,  $\chi_2$

$b$	1.5	1.5	1.5	1.5	1.5
$a$	3.390	1.846	1.199	0.858	0.653
$L_{mean}$	0.4	0.6	0.8	1.0	1.2
$L_{mod}$	0.213	0.320	0.426	0.533	0.639
$L_c$	0.2	0.3	0.4	0.5	0.6
% of $L < L_c$	26.2	26.2	26.2	26.2	26.2
$\chi_2$	0.766	0.766	0.766	0.766	0.766
$L_c$	0.8	1.2	1.6	2.0	2.4
% of $L < L_c$	91.2	91.2	91.2	91.2	91.2
$\chi_2$	0.355	0.355	0.355	0.355	0.355

interesting phenomenon is found in that the content of fibers with subcritical fiber length and the value of  $\chi_2$  are the same for all the five cases of each group. Thus the composite strength is the same if the ratios of  $L_{mean}/L_c$  and  $L_{mod}/L_c$  are the same for a given composite system no matter how large the mean fiber length, the critical fiber length and the mode fiber length are. This demonstrates that both the ratio of  $L_{mean}/L_c$  and that of  $L_{mod}/L_c$  determine the tensile strength of SFRP.

Now we consider the effect of fiber orientation distribution on the strength of non-unidirectional reinforced composites ( $\chi_1 \neq 1$ ).

Tables 2 and 3 give the effect of fiber orientation distribution on the value of  $\chi_1\chi_2$  and hence the composite strength, where  $\mu = 0.1$ ,  $A = 0.4$ ,  $L_{min} = 0$ ,  $L_{max} = \infty$ ,  $L_{mean} = 0.4$ ,  $L_{mod} = 0.213$  and  $L_c = 0.2$  mm, respectively. The results in Table 2 show that the value of  $\chi_1\chi_2$  and hence the composite strength increases with the decrease of mean fiber orientation angle and with the increase of fiber orientation coefficient. However, for the same fiber orientation coefficient,  $f_\theta$ , the values of  $\chi_1\chi_2$  are different for different mean fiber orientation angles (see Table 3); when  $f_\theta > 0$ , then the value of  $\chi_1\chi_2$  increases with the increase of mean fiber orientation angle; on the

Table 2. The effect of fiber orientation distribution on the TS of SFRP

No.	$p$	$q$	$\theta_{mean}$	$\theta_{mod}$	$f_\theta$	$\chi_1\chi_2$
1	100	0.5	1.514	$\pi/2$	-0.99	0
2	16	1	1.351	1.393	-0.88	0.013
3	8	1	1.262	1.318	-0.78	0.068
4	4	1	1.141	1.209	-0.6	0.174
5	2	1	0.982	1.047	-0.33	0.308
6	0.5	0.5	0.785	no	0	0.417
7	0.5	1	0.571	0.0	0.33	0.549
8	1	4	0.430	0.361	0.6	0.652
9	1	8	0.308	0.253	0.78	0.693
10	1	16	0.220	0.178	0.88	0.717
11	0.5	10	0.179	0.0	0.91	0.726
12	0.5	100	0.056	0.0	0.99	0.755



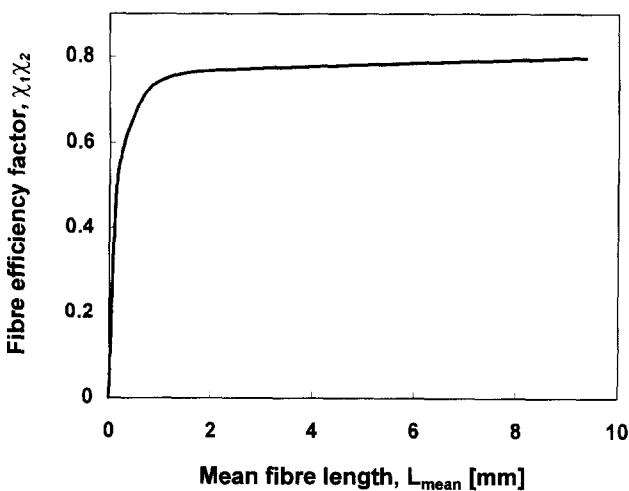
**Table 3. The effect of fiber orientation distribution on the TS of SFRP**

No.	$p$	$q$	$\theta_{\text{mean}}$	$\theta_{\text{mod}}$	$f_{\theta}$	$\chi_1\chi_2$
1	0.5	1	0.571	0.0	0.33	0.549
2	1	2	0.589	0.524	0.33	0.569
3	2	4	0.601	0.580	0.33	0.588
4	4	8	0.608	0.599	0.33	0.595
5	0.5	0.5	0.785	no	0	0.417
6	1	1	0.785	0.785	0	0.416
7	2	2	0.785	0.785	0	0.415
8	4	4	0.785	0.785	0	0.414
9	8	4	0.962	0.972	-0.33	0.351
10	4	2	0.970	0.991	-0.33	0.330
11	2	1	0.982	1.047	-0.33	0.308
12	1	0.5	0.999	1.57	-0.33	0.290

contrary, when  $f_{\theta} < 0$ , then the value of  $\chi_1\chi_2$  decreases with the increase of mean fiber orientation angle. When  $f_{\theta} = 0$ , there is almost no change in the mean fiber orientation angle and the value of  $\chi_1\chi_2$ . No direct relationship is found between the value of  $\chi_1\chi_2$  and the most probable fiber orientation angle,  $\theta_{\text{mod}}$ .

When adding the consideration of the fiber orientation distribution, since the inclined tensile strength and critical fiber length of oblique fibers are related to the inclination angle, the effect of the fiber length distribution on the TS of SFRP can be studied based on eqn (36).

Figure 10 shows the effects of the mean fiber length on the fiber efficiency factor,  $\chi_1\chi_2$ , where  $\mu = 0.1$ ,  $A = 0.4$ ,  $L_{\text{min}} = 0$ ,  $L_{\text{max}} = \infty$ ,  $L_{\text{mod}} = 0.2$  mm and  $L_c = 0.2$  mm, the parameters  $a$  and  $b$  are the same as in Figs 4 and 5 for the fiber length distribution function,  $p = 0.5$  and  $q = 10$  for the fiber orientation distribution function. It can be seen from Fig. 10 that



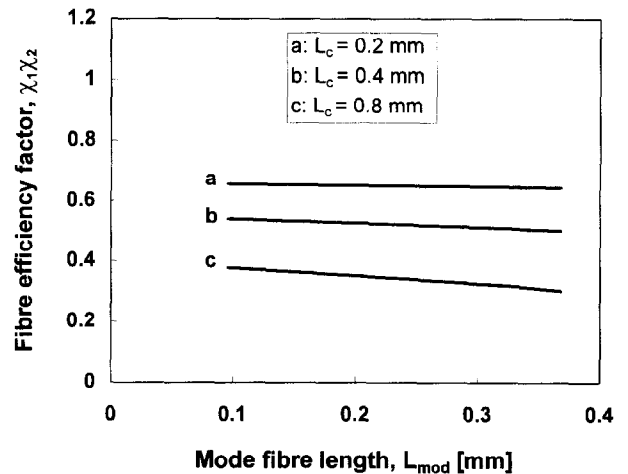
**Fig. 10.** Effect of mean fiber length,  $L_{\text{mean}}$ , on fiber efficiency factor,  $\chi_1\chi_2$ , for the case of non-unidirectional composites ( $\mu = 0.1$ ,  $A = 0.4$ ,  $L_c = 0.2$  mm,  $L_{\text{mod}} = 0.2$  mm,  $L_{\text{min}} = 0$  mm,  $L_{\text{max}} = \infty$  mm,  $p = 0.5$  and  $q = 10$  ( $\theta_{\text{mean}} = 0.179$  rad)).

the value of  $\chi_1\chi_2$  and hence the strength of composites increases rapidly with the mean fiber length at small mean fiber lengths (in the vicinity of  $L_c$ ) and approaches gradually a plateau level at large mean fiber lengths (greater than about  $5L_c$ ). This is the same as that for the case of unidirectional composites (see Fig. 5).

Figure 11 represents the effects of the mode fiber length and the critical fiber length on the value of  $\chi_1\chi_2$ . It shows that the value of  $\chi_1\chi_2$  and hence the composite strength increases with the decrease of critical fiber length; moreover, the value of  $\chi_1\chi_2$  and hence the composite strength decreases slightly with the increase of mode fiber length. These are the same as those for the case of unidirectional composites (see Figs 5, 7 and 9).

Figure 12 shows the effect of the snubbing friction coefficient on  $\chi_1\chi_2$ , where  $A = 0.4$ ,  $L_{\text{min}} = 0$ ,  $L_{\text{max}} = \infty$ ,  $L_{\text{mean}} = 0.4$  mm,  $L_c = 0.2$  mm and  $L_{\text{mod}} = 0.213$  mm, and  $p = 0.5$  and  $q = 10$  for the fiber orientation distribution function, respectively. It becomes clear that the value of  $\chi_1\chi_2$  increases slightly with the snubbing friction coefficient, which demonstrates that the snubbing friction has only a small effect on the strength of composites.

Figure 13 shows the effect of the constant  $A$  on the value of  $\chi_1\chi_2$ . It becomes clear that the value of the fiber efficiency factor,  $\chi_1\chi_2$ , decreases with the increase of the constant,  $A$ . This indicates that if the fiber flexural effect on the fracture strength of oblique fibers is smaller, then the composite strength would be higher. Otherwise, the composite strength would be lower. So the inclined tensile strength of fibers is a very useful parameter for the development of non-unidirectional SFRP.



**Fig. 11.** Effects of mode fiber length,  $L_{\text{mod}}$ , and critical fiber length,  $L_c$ , on fiber efficiency factor,  $\chi_1\chi_2$ , for the case of non-unidirectional composites ( $\mu = 0.1$ ,  $A = 0.4$ ,  $L_{\text{mean}} = 0.4$  mm,  $L_{\text{min}} = 0$  mm,  $L_{\text{max}} = \infty$  mm,  $p = 0.5$  and  $q = 10$  ( $\theta_{\text{mean}} = 0.179$  rad)).

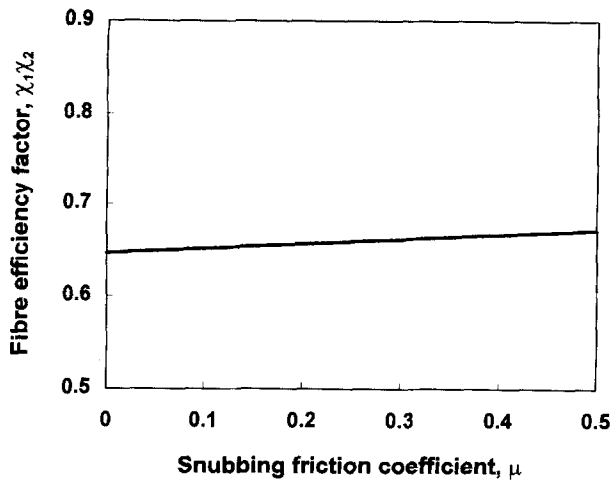


Fig. 12. Effect of snubbing friction coefficient,  $\mu$ , on fiber efficiency factor,  $\chi_1\chi_2$  ( $A = 0.4$ ,  $L_{mean} = 0.4$  mm,  $L_c = 0.2$  mm,  $L_{mod} = 0.213$  mm,  $L_{min} = 0$  mm,  $L_{max} = \infty$  mm,  $p = 0.5$  and  $q = 10$  ( $\theta_{mean} = 0.179$  rad)).

4 APPLICATION OF THE PRESENT THEORY

The purpose of developing a theoretical model is to explain and predict the experimental results. Conversely, the theoretical model should also be able to be verified by the experimental results. The present theory is applied to published experimental results.<sup>19</sup> As described in the preceding section the strength of composites can be estimated by eqns (35) and (36) for a given composite system if the fiber length distribution, the fiber orientation distribution, the critical fiber length, the snubbing friction coefficient and the inclined strength of fibers are given. The required quantities for predicting the composite strength are given in Table 4 except the snubbing friction coefficient and the inclined strength of fibers.

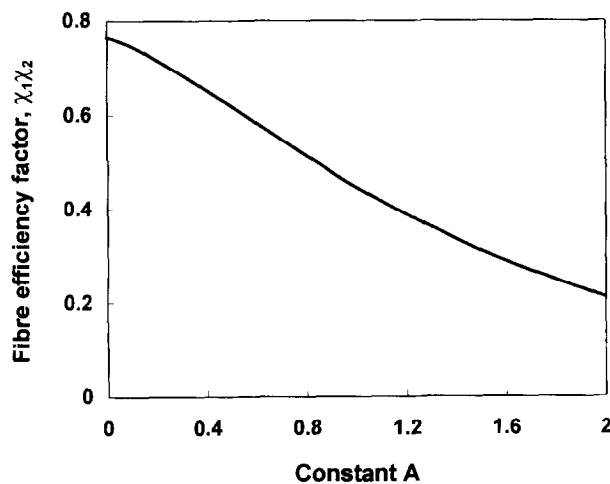


Fig. 13. Effect of constant A on fiber efficiency factor,  $\chi_1\chi_2$  ( $\mu = 0.1$ ,  $L_{mean} = 0.4$  mm,  $L_c = 0.2$  mm,  $L_{mod} = 0.213$  mm,  $L_{min} = 0$  mm,  $L_{max} = \infty$  mm,  $p = 0.5$  and  $q = 10$  ( $\theta_{mean} = 0.179$  rad)).

The snubbing friction coefficient and the inclined strength of fibers will be adjusted in the prediction of the TS of SFRP.

Since the composite strength, the fiber volume fraction and the matrix strength have been given experimentally,<sup>19</sup> then the experimental values of  $(\chi_1\chi_2)_e$  can be estimated through eqn (35) and the corresponding results are given in Table 4. Also, the mean fiber length is given,<sup>19</sup> then with eqn (10) we can get the parameter  $a$  by arbitrarily setting  $b = 1.2$  (the arbitrary setting will not result in a large error since the differently set value of  $b$  may result in the different mode fiber length; however, as shown in the preceding section (see Figs 9 and 11) the effect of mode fiber length on the TS of SFRP is very small for the case of  $L_{mean} > L_c$  and the values in Table 4 show that  $L_{mean} > L_c$ ). The parameter  $q$  can be evaluated from eqn (15) by arbitrarily setting  $p = 1$  since the fiber orientation coefficient is given, this arbitrary setting may bring about only a small error if the evaluated fiber orientation distribution from eqn (12) deviates from the real fiber orientation distribution, as given in Table 3. The snubbing friction coefficient,  $\mu$ , can be arbitrarily assumed to be 0.1 since  $\mu$  has only a very small effect on the value of  $\chi_1\chi_2$ . The constant  $A$  can be adjusted and then we can get the theoretical value of  $(\chi_1\chi_2)_t$ . The final results are given in Table 4, when varying the parameters  $a$ ,  $b$ ,  $p$  and  $q$  under the given  $L_{mean}$  and  $f_{\theta}$ , it is found that there is only a very small change in the value of  $(\chi_1\chi_2)_t$  (in order to limit the length of Table 4 it is not listed in the table) so that it can be neglected, which demonstrates that the above settings and assumption are suitable. The comparison shows that the present theory agrees well with the experimental results. The constant  $A$ , and hence the inclined strength of fibers, has a fixed value for a given composite system, and the inclined strength of fibers is related to the matrices which is consistent with that described by eqn (20).<sup>25</sup>

5 CONCLUSIONS

The effects of the fiber length distribution and the fiber orientation distribution on the strength have been studied in detail for SFRP. The results show that the strength of SFRP increases rapidly with the increase of the mean fiber length at small mean fiber lengths (in the vicinity of the critical fiber length,  $L_c$ ) and approaches a plateau level as the mean fiber length increases for the cases of large mean fiber lengths ( $> 5L_c$ ). And the composite strength increases with the decrease of critical fiber length and hence with the increase of interfacial adhesion strength and slightly with the decrease of the mode fiber length. When the ratios of  $L_{mean}/L_c$  and  $L_{mod}/L_c$  are the same for a given fiber/matrix system, the strength of SFRP will be the same no matter how large  $L_{mean}$ ,  $L_{mod}$  and

Table 4. Comparison of the present theory with the existing experimental results<sup>19</sup>

ID	Fiber strength (ksi)	Matrix strength (ksi)	Fiber volume fraction	Critical length (mm)	Mean fiber length (mm)	Fiber orientation coefficient	Composite strength (ksi)
Nylon-1	400	10.82	0.176	0.5613	0.8814	0.4333	29.22
Nylon-2	400	10.82	0.186	0.5994	1.1862	0.3461	27.70
Nylon-3	400	10.82	0.186	0.5994	0.8712	0.4095	28.40
PP-1	400	4.513	0.100	1.4554	2.4714	0.2359	8.92
PP-2	400	4.513	0.100	1.3995	2.4841	0.2029	8.42
PP-3	400	4.513	0.100	1.3995	2.4866	0.2985	10.89
PBT-1	400	4.959	0.181	0.6807	0.9144	0.3673	20.60
PBT-2	400	4.959	0.181	0.8407	0.9931	0.3407	19.98
PBT-3	400	4.959	0.181	0.6807	1.0007	0.3472	20.41

ID	$a$	$b$	$p$	$q$	$f$	$A$	$(\chi_1\chi_2)_c$	$(\chi_1\chi_2)_t$
Nylon-1	1.0813	1.2	1	2.5294	0.1	1.2	0.2884	0.2826
Nylon-2	0.7571	1.2	1	2.0586	0.1	1.2	0.2539	0.2550
Nylon-3	1.0964	1.2	1	2.3870	0.1	1.2	0.2633	0.2682
PP-1	0.3138	1.2	1	1.6175	0.1	1.7	0.1215	0.1262
PP-2	0.3118	1.2	1	1.5091	0.1	1.7	0.1090	0.1197
PP-3	0.3115	1.2	1	1.8510	0.1	1.7	0.1707	0.1425
PBT-1	1.0346	1.2	1	2.1611	0.1	1.3	0.2284	0.2237
PBT-2	0.9370	1.2	1	2.0335	0.1	1.3	0.2199	0.2082
PBT-3	0.9284	1.2	1	2.0637	0.1	1.3	0.2258	0.2202

$L_c$  are. In general the strength of composites increases with the increase of fiber orientation coefficient,  $f_\theta$ , and the decrease of mean fiber orientation angle; however, when the fiber orientation coefficients are the same, the strength of composites increases with the increase of mean fiber orientation angle for  $f_\theta > 0$  and increases with the decrease of mean fiber length for  $f_\theta < 0$ . No direct relationship is found between the strength of SFRP and the most probable fiber orientation angle. The effect of the snubbing friction between fiber and matrix on the strength of SFRP is small. The inclined tensile strength of fibers has a great effect on the strength of composites. The present theory is successfully applied to the existing experimental results.

#### ACKNOWLEDGEMENT

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